THE DICE GAME OF SUCKERS

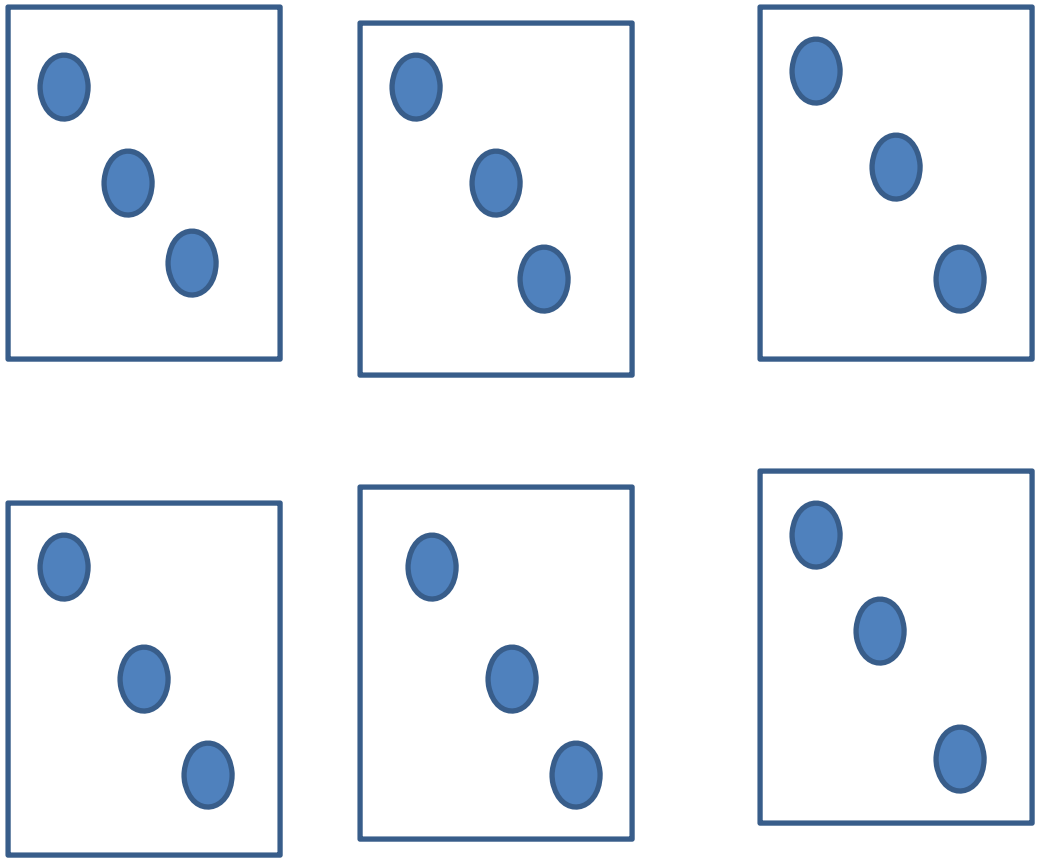
(The Intransitive Dice)

By Steve Edgell

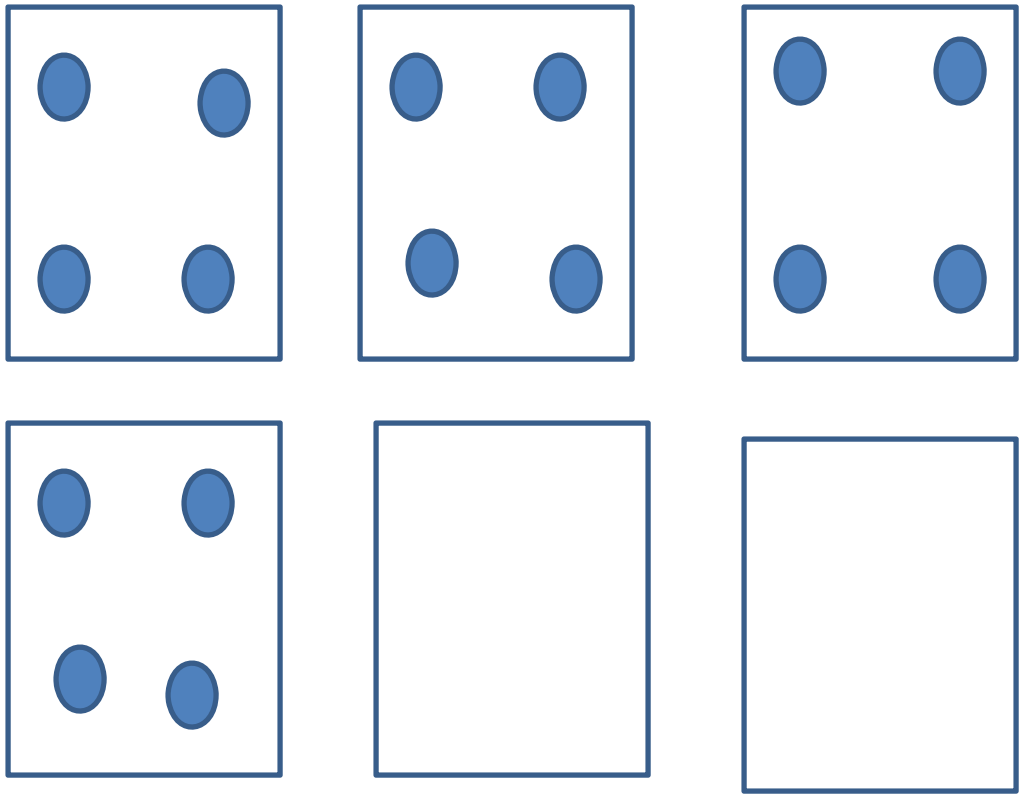
The Dice

- OK so all dice games are games for suckers, but for this one, it is especially true.
- There are 4 dice. We will call them die A, die B, die C, and die D
  - They are fair in that when rolled each side is equally likely.
  - However, they have a different number of spots on each of their 6 sides than do regular dice.
- Let’s look at the dice:

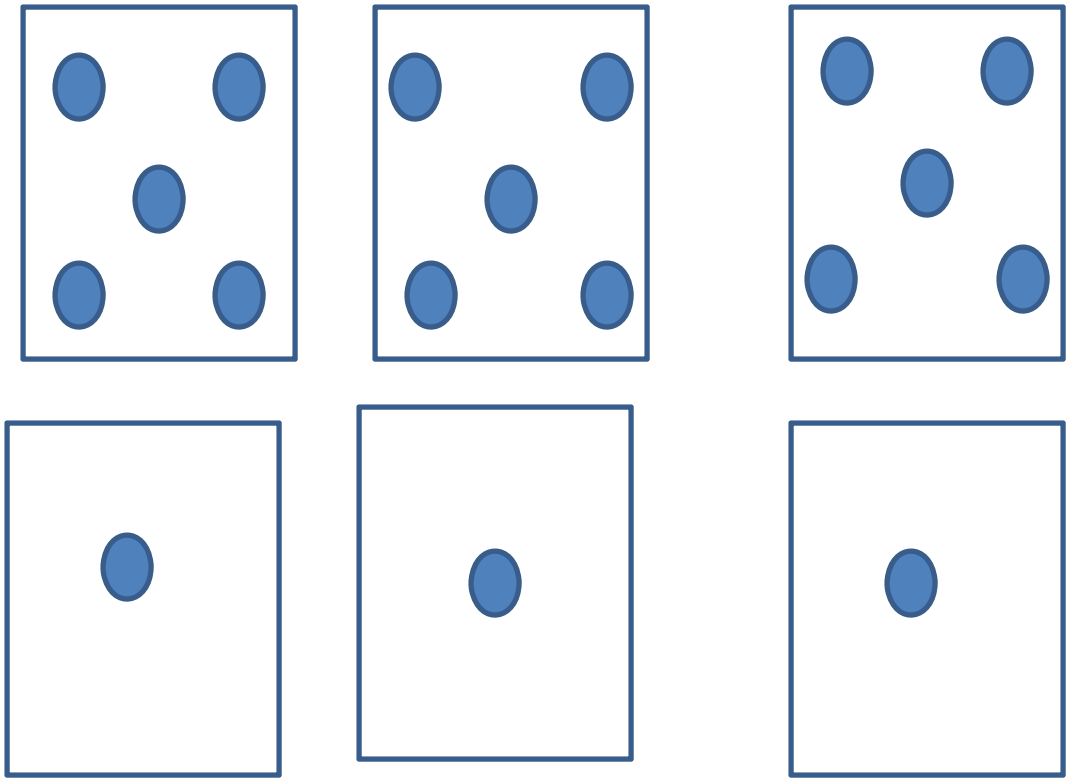
Die A



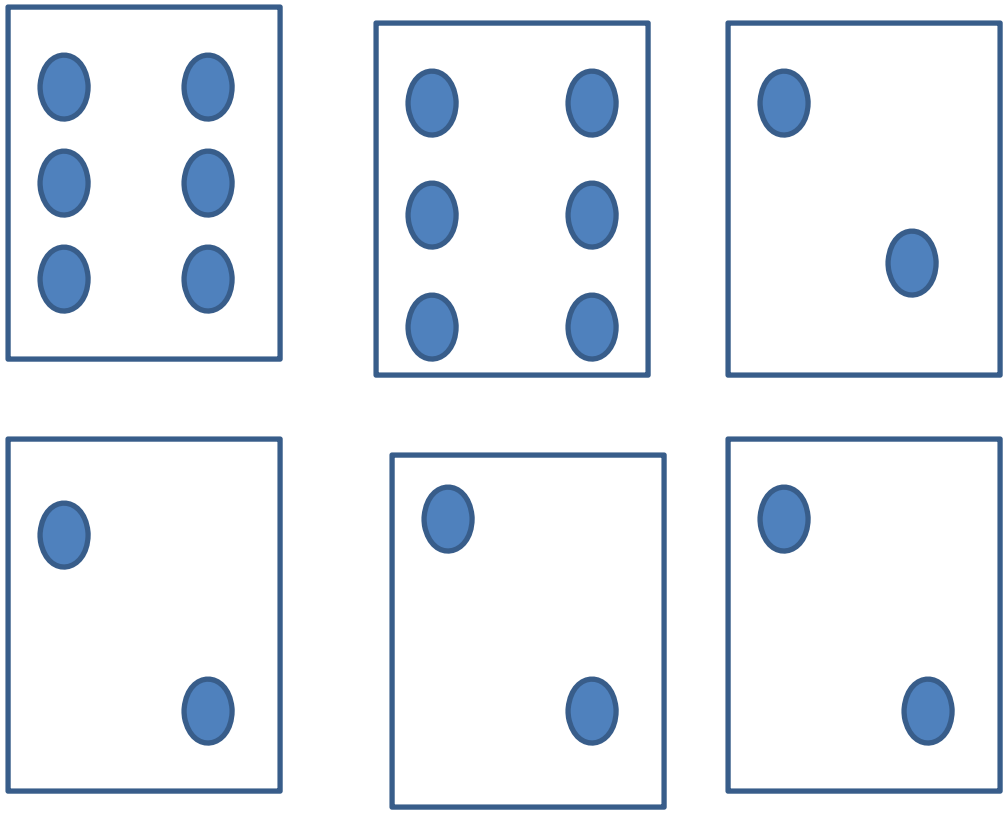
Die B



Die C



Die D



The Dice Table

- It is a good idea to copy down this table so you can refer to it as you proceed.
- This is a summary of the four dice showing what's on the 6 sides:

A	3	3	3	3	3	3
B	4	4	4	4	0	0
C	5	5	5	1	1	1
D	6	6	2	2	2	2

The Game

- You pick from the 4 dice the one you wish.
- Then I'll pick from the remaining 3.
- We each roll our die—high number wins
  - Note: that no two dice have the same number so ties are impossible.
- Any time you like, we will both put our die back and again you can chose from the 4.
  - You can always have the die you want!

Let's Play

- OK. Suppose you take die A.
  - You know you will get a 3 on every roll!
- I take die B.
  - With probability 4/6=2/3, I will roll a 4 and with probability 1/3, I will roll a 0.
    - If I roll 4, I win; if I roll a 0, you win
  - So I win with probability 2/3. Not good for you.
- So you call for a new choice and you take die B
  - Well, I take die C.

What is the probability die C beats die B?

- Die B, as before, gets a 4 with p=2/3 and a 0 with p=1/3
- Now die C gets a 5 with p=1/2 and a 1 with p=1/2
- What is the probability C beats B
- Stop! Don’t go on to the next slide until you figure it out!

Did you figure it out or are you cheating and going on without figuring it out yourself?

Probability C beats B

- P(C wins)=
- P[C rolls a 5 OR (C rolls a 1 AND B rolls a 0)]=
- P(C rolls a 5) + P(C rolls a 1 AND B rolls a 0)
  - C either rolls a 5 or C rolls a 1, BUT NOT BOTH
- P(C rolls a 5) + P(C rolls a 1) P(B rolls a 0)
  - The two dice are independent
- 1/2 + (1/2) (1/3) = 1/2 + 1/6 = 2/3

OK

- So again I will win with probability 2/3!
- So you want die C!
- Fine by me, because I take die D.
  - You saw that one coming, didn't you.
- Before going on to the next slide, you figure the P(D beats C).

P(die D beats die C)

- P(D wins) =
- P[D rolls 6 OR (D rolls 2 AND C rolls 1)]
- P(D rolls 6) + P(D rolls 2) P(C rolls 1)
  - Same reasons
- 1/3 + (2/3) (1/2) = 2/3
- Again I win with p=2/3!
- So you've got the game figured out, and you take die D! BUT…

P(A beats D)

- If you take D, I take A. P(A wins)=
- P(D rolls 2) = 2/3
- Again I win with P=2/3!!

- B is better than A, C is better than B, D is better than C, and A is better than D
  - Such a circular relationship is called intransitive.