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Exploring mathematics problems prepares children to learn from instruction

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ABSTRACT

Both exploration and explicit instruction are thought to benefit learning in many ways, but much less is known about how the two can be combined. We tested the hypothesis that engaging in exploratory activities prior to receiving explicit instruction better prepares children to learn from the instruction. Children (159 second- to fourth-grade students) solved relatively unfamiliar mathematics problems (e.g., $3 + 5 = 4 + \square$) before or after they were instructed on the concept of mathematical equivalence. Exploring problems before instruction improved understanding compared with a more conventional "instruct-then-practice" sequence. Prompts to self-explain did not improve learning more than extra practice. Microgenetic analyses revealed that problem exploration led children to more accurately gauge their competence, attempt a larger variety of strategies, and attend more to problem features—better preparing them to learn from instruction.

Introduction

Some theories of learning and development focus on how much children learn through exploration and self-discovery of their environment without explicit instruction from a more knowledgeable person ("exploration"; Hirsh-Pasek, Golinkoff, Berk, & Singer, 2009; Piaget, 1973; Schulz & Bonawitz, 2007; Sylva, Bruner, & Genova, 1976). Other theories focus on how children learn through guidance and instruction from more knowledgeable others such as parents and teachers ("explicit instruction"; Csibra & Gergely, 2009; Kirschner, Sweller, & Clark, 2006; Tomasello, Carpenter, Call, Behne, & Moll, 2005; Vygotsky, 1978). Both exploration and explicit instruction are thought to benefit learning in

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numerous ways. For example, allowing learners to explore a new environment or topic area may increase their motivation, encourage broad hypothesis testing, and improve depth of understanding (e.g., Bonawitz et al., 2011; Piaget, 1973; Sylva et al., 1976; Wise & O'Neill, 2009). At the same time, children learn extensively from social partners, and teaching children new information directly can lessen the burden on cognitive resources and support the development of accurate knowledge (Kirschner et al., 2006; Klahr & Nigram, 2004; Koenig & Harris, 2005; Sweller, van Merrienboer, & Paas, 1998; Tomasello et al., 2005).

We evaluated whether the benefits of both exploration and explicit instruction can be combined to improve children's learning of a mathematical concept. In the current study, children were given explicit instruction on a mathematical concept. This instruction was combined with a problem-solving activity. Some children were given the problem-solving activity as an exploratory activity, which we defined in this context as an experience in which one has received little, if any, specific instruction on the task and, therefore, must explore the parameters of the experience for oneself (Bonawitz et al., 2011). After exploration, these children received explicit instruction. Other children received explicit instruction first, followed by problem-solving practice. Beginning with instruction constrains and guides students' subsequent problem solving (Bonawitz et al., 2011; Sweller et al., 1998). Thus, the same problem-solving activity was used to promote exploration prior to instruction or practice after instruction. We reasoned that combining the strengths of exploration with subsequent explicit instruction would better prepare children to learn from the instruction (cf. Dewey, 1910; Schwartz, Lindgren, & Lewis, 2009). By comparing learning in this condition with a more conventional "instruct-then-practice" condition, we sought to reveal how exploration changes how children attend to, process, and ultimately learn new information. Thus, the current work contributes to a theoretical framework that seeks to understand the mechanisms by which different learning experiences can be combined and how they affect children's knowledge growth.

Exploratory activities and improving learning from instruction

Parents and teachers often use a "tell-then-practice" approach to teaching (Eisenberg et al., 2010; Hiebert et al., 2003; Roelofs, Visser, & Terwel, 2003). First children are told the important information, and then they are asked to practice using this information. This method helps children to quickly hone their attention to the most important information and, therefore, reduces the burden on cognitive resources (Bonawitz et al., 2011; Clark, 2009; Sweller et al., 1998). As a result, children spend less time on trial and error and reduce their use of erroneous beliefs or behaviors, instead practicing an appropriate way of thinking or acting in a domain (e.g., Eisenberg et al., 2010; Rittle-Johnson, 2006). Critically, without accurate instructional guidance, children often fail to discover correct information (e.g., Alfieri, Brooks, Aldrich, & Tenenbaum, 2011; Klahr & Nigram, 2004; Koenig & Woodward, 2010; Mayer, 2004).

Despite the demonstrated benefits of explicit instruction from more knowledgeable others, instruction can lead to overly narrow knowledge structures and problem-solving behaviors. For example, explicit instruction may lead children to stop searching for additional ideas or information (e.g., when playing with a toy), as they perceive the instruction as having provided all that they need to learn (Bonawitz et al., 2011). Indeed, explicit instruction can reduce problem-solving success, as children try fewer means to solve a task after a demonstration by an adult than after free play (Sylva et al., 1976). In addition, children may harbor misconceptions, irrelevant intuitions, or false beliefs about their own understanding when they are taught new information, which often hinders their understanding of the new information (Carey, 1985; Eryilmaz, 2002; Hartnett & Gelman, 1998; McNeil & Alibali, 2005; Vamvakoussi & Vosniadou, 2004). Explicit instruction can also lead individuals to have greater feelings of fluency with the material and become overly confident in their level of understanding (Bjork, 1994; Gerjets, Scheiter, & Catrambone, 2004).

Engaging children in exploratory experiences prior to providing them with explicit instruction may help to allay the potential pitfalls of using explicit instruction alone. Exploratory activities may alert children to the need to make sense of the experiences they are encountering (Dewey, 1910; Hiebert & Grouws, 2007; Schwartz & Bransford, 1998; Wise & O'Neill, 2009). In particular, exploratory experiences may prompt children to begin to wrestle with the similarities and differences between these experiences and their prior knowledge, including noticing inconsistencies in their prior knowledge (i.e., inducing cognitive conflict; Carey, 1985; Eryilmaz, 2002). Indeed, these experiences may help to dispel children's illusions of competence by helping children to realize their limited understanding of the topic (Bjork, 1994). Despite the inherent difficulty of exploration, this process could lead children to search for new information and begin to revise their schemas, which should aid them in encoding new instruction in a more meaningful and relevant way (a "desirable difficulty"; Bjork, 1994; see also Carpenter, Franke, & Levi, 2003; Dewey, 1910; Duffy, 2009). Thus, even if limited, prior exploratory experiences could better align instruction with children's capabilities to understand (i.e., could place new instruction within their zone of proximal development; Vygotsky, 1934/1987; see also Hiebert & Grouws, 2007; Schwartz, Sears, & Chang, 2007). Children would be better prepared to learn from instruction (Dewey, 1910).

Recent evidence supports the idea that people benefit from exploratory activities prior to instruction. For example, college students who initially explored a set of examples learned more deeply from a lecture on cognitive psychology principles than students who simply summarized a relevant text passage before the lecture (Schwartz & Bransford, 1998). Similarly, ninth-grade students who explored sets of data before instruction and practice with descriptive statistics were better able to learn from new instructional resources than students who received extended explicit instruction followed by practice (Schwartz & Martin, 2004). These benefits of initial exploratory practice were not apparent on traditional measures of memory or on success in solving familiar problem types; rather, they were apparent on complex problems requiring new insights. Findings such as these led Schwartz and colleagues to conclude that initial exploratory practice prepares people to learn more deeply from instruction compared with providing instruction first (Schwartz & Bransford, 1998; Schwartz & Martin, 2004; Schwartz et al., 2009; Sears, 2006).

Schwartz and colleagues' proposal suggests that the common approach of first providing explicit instruction and then having people practice using the instructional information is not optimal. However, despite initial evidence in support of this idea, more empirical evidence is needed. First, in Schwartz and colleagues' previous studies, the exploratory practice activities and the outcome measures that distinguish between conditions are complex and rarely used in schools or homes. It remains to be seen whether simpler activities can produce similar benefits. Second, these studies do not explicitly compare the exploratory learning (plus instruction) conditions with a control condition in which people are given the exact same learning activities in reverse order. Therefore, the effects of exploration cannot be isolated from the potential benefits of the learning activities themselves (e.g., contrasting datasets). Third, it is unknown whether children would benefit from initial exploratory experiences as well as the adolescents and adults in previous studies. Exploration can place heavier demands on working memory than explicit instruction (Kirschner et al., 2006), and working memory capacity develops with age, with children demonstrating lower capacity than adolescents and adults (Gathercole, Pickering, Ambridge, & Wearing, 2004). Finally, the mechanisms by which learning is improved have not been empirically examined. In general, lack of experimental evidence is cited as a primary reason for the persistence of debates on the timing of explicit instruction (Mayer, 2009). The need for further research in this area is quite important.

We propose that simply reversing the order of explicit instruction and problem solving should benefit children's learning. Specifically, we compared conditions in which children completed the exact same tasks but used problem solving with feedback as exploration in one condition (prior to instruction) and as practice in another condition (after instruction), thereby keeping the learning materials the same across conditions. We anticipated that the simple experience of exploring unfamiliar problems on one's own should (a) help children to better gauge their understanding of the underlying concept (or lack thereof) and (b) challenge children to try new ways to solve problems, thereby helping them to notice important problem features. As a result, this more accurate assessment of their abilities and differentiated prior knowledge should help individuals to process subsequent instruction at a deeper level, improving conceptual knowledge acquisition and retention. We expected these effects to occur with elementary school children and on traditional measures of understanding, both immediately and over a delay, and we gathered empirical evidence for these proposed mechanisms.

Self-explanation as an exploration tool

If exploratory activities help children to build relevant prior knowledge before explicit instruction, then activities that help children to notice the most important information, such as self-explaining, may also benefit learning. Self-explanation prompts ask learners to try to explain *why* correct content is true, encouraging learners to actively manipulate, link, and evaluate information, and have been shown to improve learning across a variety of domains (e.g., Chi, de Leeuw, Chiu, & LaVancher, 1994; Honomichl & Chen, 2006; Rittle-Johnson, 2006; Wellman, 2011). However, the benefits of self-explanation for learning do not always outweigh the benefits of other learning activities that take a comparable amount of time to complete (e.g., additional problem-solving practice; Matthews & Rittle-Johnson, 2009). We examined the impact of self-explanation in conjunction with exploratory activities, reasoning that self-explanation may bolster learning from exploration. However, the exploratory experience itself may be sufficient to help children prepare to learn from instruction; thus, self-explanation might not significantly improve performance compared with solving additional problems, demonstrating a possible boundary condition for the utility of self-explanation.

The current study

In this study, we investigated whether initial exploratory experiences and self-explanation prompts helped children to learn from a brief explicit instruction. Through a series of microgenetic analyses, we also examined potential reasons why different types of instructional experiences may alter learning.

Children were instructed on the concept of *mathematical equivalence*—the concept that the two sides of the equal sign represent the same quantity. Understanding mathematical equivalence is an indicator of children's flexibility in representing and using basic arithmetic ideas and is an important prerequisite for understanding algebra (Carpenter et al., 2003; Kieran, 1992; Knuth, Stephens, McNeil, & Alibali, 2006; Rittle-Johnson, Taylor, Matthews, & McEldoon, 2011). Unfortunately, based on early experiences in solving problems where the equal sign is at the end of the problem (e.g., $3 + 5 = \square$), children often incorrectly infer that the equal sign is a symbol that means to "sum" or "get the answer" (e.g., Carpenter et al., 2003). Thus, when solving problems involving operations on both sides of the equal sign (e.g., $3 + 5 = \square + 2$, often called a mathematical equivalence problem), children often add the numbers to the left of the equal sign (e.g., answering "8") or add all of the numbers (e.g., answering "10"), consistent with their misconceptions about the meaning of the equal sign (e.g., McNeil & Alibali, 2005; Perry, Church, & Goldin Meadow, 1988). Second- to fourth-grade students have the computational skills to solve arithmetic problems but generally have little prior experience in solving equivalence problems.

In the current study, children solved mathematical equivalence problems with accuracy feedback either before or after receiving brief explicit instruction on the concept of mathematical equivalence. During problem solving, children either self-explained or solved additional problems to equate the time spent on the task. Thus, each child received one of four tutoring conditions based on a crossing of these two factors (i.e., instruction order and self-explanation condition).

Children's conceptual and procedural knowledge of mathematical equivalence was assessed prior to the individual tutoring intervention, immediately following the intervention, and again after an approximately 2-week delay using a detailed assessment developed in previous work (i.e., Rittle-Johnson et al., 2011). *Conceptual knowledge* consists of abstract or generic ideas generalized from particular instances, including knowledge of problem structures. *Procedural knowledge* is knowledge of action sequences to solve problems (e.g., Greeno, Riley, & Gelman, 1984; Hiebert & Wearne, 1996; Rittle-Johnson, Siegler, & Alibali, 2001). Although these are distinct types of knowledge, they lie along a continuum, often developing in an iterative fashion (Rittle-Johnson & Schneider, in press; Rittle-Johnson et al., 2001). Procedural knowledge items were similar to the problems used during the intervention and, thus, tapped the ability to use or modify solution procedures learned during the intervention. Conceptual knowledge items tapped both knowledge of the equal sign and knowledge of equation structures, measured in a variety of ways and in formats that were not practiced during the intervention. Unlike a majority of past assessments of mathematical equivalence knowledge, which have focused exclusively on equations with operations on both sides of the equal sign and/or explicit knowledge of the meaning of the equal sign, our assessment also captured earlier emerging knowledge such as comfort with operations on only the right side of the equal sign (e.g., 8 = 3 + 5) and comfort with no operations (e.g., 8 = 8) (see Rittle-Johnson et al., 2011, and Matthews, Rittle-Johnson, McEldoon, & Taylor, 2012, for details on this measure).

Additional measures were collected during the intervention (e.g., understanding of equivalence, subjective ratings of understanding) to assess the progression of learning and related factors.

We hypothesized that reversing the traditional tell-then-practice approach would benefit learning in children. Indeed, rather than inducing too great of a cognitive load (as would be the case if children never received instruction), we anticipated that asking children to solve problems before instruction would produce a desirable difficulty (Bjork, 1994), preparing them to learn from instruction. We also anticipated that self-explanation prompts would enhance this effect.

Specifically, we expected that children would encounter difficulty during the exploratory problem-solving period, likely using incorrect strategies more often than they would if they solved the problems after instruction. However, in part because children were given accuracy feedback, this difficulty should lead them to more accurately assess their understanding (Bjork, 1994), use a greater variety of strategies (e.g., Alibali, 1999; Fyfe, Rittle-Johnson, & DeCaro, in press), and encode important features of the problems (Schwartz et al., 2007; Siegler & Chen, 1998). Thus, children who solve problems before explicit instruction may perform worse during problem solving but demonstrate greater conceptual understanding at posttest. Moreover, because prior work suggests that the benefits of exploration occur on measures of deeper understanding that reflect more integrated knowledge beyond simple procedures (Schwartz et al., 2009; see also Hiebert & Grouws, 2007), we expected the benefit of exploring problems prior to instruction to selectively occur on the conceptual knowledge measure. We expected children across groups to make similar improvements in procedural knowledge.

Methods

Participants

Consent was obtained from 229 second- to fourth-grade students at a suburban public school serving a middle-class population. A pretest was given to identify children who did not already demonstrate a high level of mathematical equivalence knowledge so that differences in learning between conditions could be detected. Children scoring below 80% on this pretest were selected for the study and randomly assigned to experimental conditions. Data from 5 of these children were excluded (2 because their instruction session was substantially interrupted, 2 because they had diagnosed learning disabilities, and 1 because he was not available to participate in the intervention). The final sample ($N = 159$, 56% girls and 44% boys) consisted of 77 second-graders, 56 third-graders, and 26 fourth-graders. The average age was 8.5 years (range = 7.3–10.8). Approximately 18% of the participants were ethnic minorities (10% African American, 6% Asian, and 2% Hispanic). These demographic characteristics did not differ significantly by condition. An analysis of the second- to fourth-grade textbooks used at the school (Greenes et al., 2005) indicated that, of all instances of the equal sign, operations were present on both sides of the equal sign only 1 to 6% of the time and never included a definition of the equal sign (Rittle-Johnson et al., 2011). Thus, equations with operations on both sides of the equal sign were relatively unfamiliar to these students.

Design

Children completed a pretest, brief tutoring intervention, immediate posttest, and delayed retention test. Children were randomly assigned to one of four intervention conditions in a 2 (Order: solve–instruct or instruct–solve) $\times$ 2 (Explanation Condition: extra practice or self-explanation) design: solve–instruct with extra practice ($n = 40$: 21 second-graders, 12 third-graders, 7 fourth-graders), solve–instruct with self-explanation ($n = 39$: 19 second-graders, 15 third-graders, 5 fourth-graders), instruct–solve with extra practice ($n = 38$: 18 second-graders, 14 third-graders, 6 fourth-graders), and instruct–solve with self-explanation ($n = 42$: 19 second-graders, 15 third-graders, 8 fourth-graders).

Materials

Assessment

The mathematical equivalence assessment was adapted from past research (e.g., Carpenter et al., 2003; Matthews et al., 2012; McNeil & Alibali, 2004, 2005; Rittle-Johnson, 2006; Rittle-Johnson et al., 2011). Two parallel forms of the assessment were used, with Form 1 administered at pretest and Form 2 administered at posttest and retention test. The two forms differed primarily in the specific numbers presented in the items. The assessment included conceptual and procedural knowledge scales (10 points each). *Conceptual knowledge* items assessed two key concepts of mathematical equivalence: (a) the meaning of the equal sign as a relational symbol and (b) the structure of equations, including the validity of equations with no operations or with operations on only the right side or both sides of the equal sign (see Table 1). *Procedural knowledge* problems were eight mathematical equivalence problems, which had operations on both sides of the equal sign (e.g., 4 + 5 + 8 = □ + 8), and two easier but nonstandard problems (e.g., 7 = □ + 5). Far transfer was assessed at retention test with seven items intended to tap a higher level of conceptual thinking (e.g., “17 + 12 = 29 is true. Is

Table 1
Conceptual knowledge assessment items.

Concept	Item	Scoring criteria
Structure of equations	1. Correct encoding: Reproduce three equivalence problems, one at a time, from memory after a 5-s delay	1 Point if child put numerals, operators, equal sign, and blank in correct positions for all three problems (position, but not value of, the numerals must be correct)
	2. Recognize correct use of equal sign in multiple contexts:	
	(a) Indicate whether seven equations in nonstandard formats, such as 8 = 5 + 3 and 5 + 3 = 3 + 5, are true or false	1 Point if 75% of equations correctly identified as “true” or “false”
	(b) Explain why two equations are true	1 Point per explanation if child shows through words or mathematics that both sides of the equation are the same
Meaning of equal sign	1. Define the equal sign	1 Point if defined relationally (e.g., “both sides are the same”)
	2. Identify the pair of numbers from a list that is equal to another pair of numbers (e.g., 6 + 4)	1 Point if identified correct pair of numbers
	3. Identify the symbol from a list that, when placed in the empty box (e.g., “10 cents □ one dime”), will show that the two sides are the same amount	1 Point if chose the equal sign
	4. Rate definitions of the equal sign: Rate three definitions (two fillers) as “good,” “not good,” or “don’t know”	1 Point if rated the statement “The equal sign means two amounts are the same” or ”The equal sign means the same as” as a good definition
	5. Which (of the above) is the best definition of the equal sign?	1 Point if chose the relational definition (see above)
	6. Define the equal sign in the context of a money-related question (e.g., 1 dollar = 100 pennies)	1 Point if defined relationally

17 + 12 + 8 = 29 + 8 true or false? How do you know?''). However, accuracy on these items was quite low, and no differences were found across conditions, so we do not discuss this far transfer measure further. A brief version of Form 1 (three conceptual knowledge items and two procedural knowledge items) was used as a midtest during the tutoring intervention to assess the progression of knowledge change during the intervention.

Intervention problem solving

Intervention materials were presented on a laptop computer using E-Prime 2.0 (Psychology Software Tools). All instruction was scripted and well practiced so that all experimenters gave the same instruction to all of the children. During the problem-solving block, children saw mathematical equivalence problems one at a time on the computer screen and were asked to try to figure out the number that went in the box to make the number sentence true. Children were given pencil and paper and told that they could use these to help them solve the problems. When they had an answer, they typed this answer on a number pad connected to the computer. After each problem, children were asked to report their strategy use. Then they were given accuracy feedback. Specifically, children were told whether their answer was correct or incorrect, and the correct answer was displayed on the computer screen and read aloud.

Problems increased in difficulty to help elicit relevant prior knowledge (cf. Carpenter et al., 2003). The first problem in a set was a three-operand problem (e.g., 10 = 3 + □) that children often solve correctly, followed by two four-operand problems (e.g., 3 + 7 = 3 + □, 3 + 7 = □ + 6). The next three problems were five-operand problems with a repeated addend on the two sides of the equation (e.g., 5 + 3 + 9 = 5 + □).

We manipulated what children did during the problem-solving block. Children in the *extra practice* condition solved two sets of six problems each to attempt to control for the amount of time other children spent self-explaining. Problems in the second set were isomorphic to those in the first set. Children in the *self-explanation* condition solved one set of six problems and were given self-explanation prompts after they solved each problem using the same procedure as in previous work (e.g., Matthews & Rittle-Johnson, 2009; Rittle-Johnson, 2006). Specifically, after solving each problem and reporting their strategy use, children were shown one correct answer and one typical incorrect answer. Children were asked to explain how a child might have gotten each answer and then why the answer was correct or incorrect. ''Why'' prompts were used in addition to ''how'' prompts to help children discriminate simple problem-solving procedures from the deeper conceptual responses associated with the benefits of self-explaining (Chi et al., 1994). Explaining both correct and incorrect answers helps children to distinguish correct and incorrect ways of thinking (Siegler, 2002).

Intervention instruction

During instruction (adapted from Matthews & Rittle-Johnson, 2009), children were briefly taught about the relational meaning of the equal sign. Specifically, number sentences were shown on the computer screen, and the experimenter gave scripted explanations of both the structure of these number sentences (i.e., that there are two sides) and the explicit meaning of the equal sign (i.e., that the equal sign means that both sides are equal or the same). For example, for the first number sentence, 3 + 4 = 3 + 4, children were told, ''There are two sides to this problem, one on the left side of the equal sign [sweeping gesture under left side] and one on the right side of the equal sign [sweeping gesture under right side]. . . . What the *equal sign* [pointing] means is that the things on both sides of the equal sign are equal or the same [sweeping hand back and forth].'' Then the meaning of the equal sign was reiterated with four other number sentences of various sorts (e.g., 4 + 4 = 3 + 5; 3 + 4 = □; 5 + 4 + 3 = 5 + □). Children were often prompted with questions and asked to point to the two sides of the problem to ensure that they were attending to instruction. Solution procedures were not discussed.

Additional measures

Children were asked two questions to assess their *subjective ratings of understanding*. Specifically, children were shown a new mathematical equivalence problem (5 + 2 + 3 = 7 + □) and asked whether they thought they could solve it correctly, and then they were asked whether they thought they understood what the equal sign means. Response options for both items were "yes," "maybe," and "probably not," which were later converted to a 3-point scale and averaged. Two other measures were administered but were not relevant for the current study and, therefore, are not discussed further: a self-report on learning and performance goals for mathematics (adapted from Elliot, 1999) and a measure of working memory capacity (backward digit span; Wechsler, 2003).

Procedure

Children completed the written pretest in their classrooms in one 30-min session. Children who scored below 80% on the pretest completed a one-on-one tutoring intervention and immediate posttest in one session lasting approximately 45 min. This session occurred at least 1 day after the pretest. The tutoring intervention consisted of two components, a problem-solving block and an instruction block. The order in which students completed these blocks varied. Specifically, children given the *instruct–solve* order received instruction first, followed by the problem-solving block. Children given the *solve–instruct* order did the opposite, completing the problem-solving block first, followed by instruction. The problem-solving block was audio- and video-recorded.

The midtest and subjective ratings of understanding measure were administered between the two blocks, and the subjective ratings of understanding measure was administered again at the end of the intervention. Approximately 2 weeks after the tutoring intervention, children completed the written retention test in group sessions.

Coding

On the conceptual knowledge assessment, each item was scored using the criteria in Table 1. Two raters who were blind to the experimental conditions independently coded 20% of the items requiring an explanation, and interrater agreement was high (kappas = 89–96%). On the procedural knowledge assessment, answers to problems were scored as correct if they came within 1 of the correct answer to reduce false negatives due to arithmetic errors. Scores were converted to percentages correct.

Children's strategy reports were transcribed and coded by trained raters. Two raters independently coded 20% of the strategy reports, and interrater agreement was high (kappa = 80%). We also coded children's self-explanations during the intervention for whether children mentioned the concept of equivalence (e.g., "They both have to equal the same thing"), and interrater agreement on 20% of explanations was high (kappa = 89%).

Treatment of missing data

On the day of the retention test, 6 participants (3.7% of the sample) were absent from school (1 in the solve–instruct with extra practice condition, 3 in the instruct–solve with extra practice condition, and 2 in the instruct–solve with self-explanation condition). These participants did not differ from the other participants on the pretest. Their missing data were replaced using a multiple imputation technique because multiple imputation leads to more precise and unbiased conclusions than does case-wise deletion (Peugh & Enders, 2004; Schafer & Graham, 2002). We used the expectation maximization (EM) algorithm for maximum likelihood estimation via the missing value analysis module of SPSS, as recommended by Schafer and Graham (2002). Children's missing scores were estimated from all nonmissing values on continuous variables that were included in the analyses presented below. Analyses using a case-wise deletion approach yielded the same basic findings.

Results

Pretest

At pretest, children who were included in the intervention answered a minority of procedural and conceptual items correctly (M = 47% correct, SD = 27, and M = 36% correct, SD = 19, respectively). For

Table 2
Mean percentages correct on procedural and conceptual knowledge assessments as a function of order and explanation condition at posttest and retention test.

Assessment	Instruct–solve condition		Solve–instruct condition	
	Extra practice	Self-explain	Extra practice	Self-explain
Conceptual knowledge				
Posttest	59 (28)	60 (23)	64 (22)	64 (24)
Retention test	62 (26)	63 (21)	65 (22)	69 (21)
Procedural knowledge				
Posttest	74 (33)	73 (30)	70 (27)	69 (32)
Retention test	76 (31)	76 (28)	75 (27)	78 (28)

Note: Standard deviations are in parentheses.

example, most children were successful on equations with a single addition operation on the right side of the equal sign. As expected, neither conceptual nor procedural knowledge differed by order or explanation condition at pretest (Fs < 1).

Posttest and retention test

We expected that children who solved problems prior to instruction would show better conceptual knowledge—indicative of a deeper level of understanding—than children who solved problems after instruction. We also investigated whether self-explanation prompts improved learning more than solving additional problems, particularly when used prior to conceptual instruction. We expected similar effects of order and explanation condition across both posttest and retention test. To evaluate these hypotheses, we conducted 2 (Order: instruct–solve or solve–instruct) $\times$ 2 (Explanation Condition: extra practice or self-explanation) $\times$ 2 (Time: posttest or retention test) analyses of covariance (ANCOVAs), with order and explanation condition as between-participants factors and time as a within-participants factor. Conceptual and procedural knowledge pretest scores, as well as children's age at pretest, were included in all analyses as covariates to control for prior knowledge. In preliminary analyses, we explored whether effects of condition depended on pretest knowledge or age (e.g., whether either pretest measure interacted with order or explanation condition), but they did not, so these terms were not included in the final models. Effect sizes are reported using partial eta-squared.

Procedural knowledge

First, we verified that condition did not affect procedural knowledge. There was a marginal effect of time on procedural knowledge, $F(1,152) = 3.07$, $p = .082$, $\eta_p^2 = .02$; children's percentage correct on the procedural knowledge items generally improved from posttest (M = 72%, SE = 2) to retention test (M = 76%, SE = 2) (see Table 2). However, there were no effects of order or explanation condition or any interactions (Fs < 1). Children across all experimental conditions demonstrated greater procedural knowledge after the tutoring intervention, and neither the order of instruction nor the explanation condition differentially affected performance.

Conceptual knowledge

Next, we considered differences in conceptual knowledge. A main effect of order was found for conceptual knowledge, $F(1,152) = 5.07$, $p = .026$, $\eta_p^2 = .03$. As shown in Fig. 1, children given the solve–instruct order scored higher on the conceptual knowledge measure (M = 67%, SE = 2) than children given the instruct–solve order (M = 60%, SE = 2). There was no effect of explanation condition or an Order $\times$ Explanation Condition interaction (Fs < 1), indicating that self-explanation did not help children's conceptual knowledge beyond additional problem-solving practice alone for either instructional order (see Table 2). Finally, there was a main effect of time on conceptual knowledge,

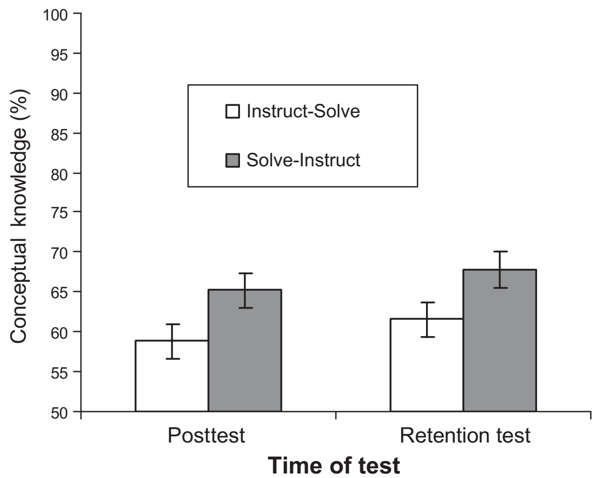

Fig. 1. Conceptual knowledge (estimated marginal means) at posttest and retention test as a function of instruction order. Error bars represent standard errors.

$F(1,152) = 8.75$, $p = .004$, $\eta_p^2 = .05$, indicating that conceptual knowledge scores improved from posttest ($M = 62\%$, $SE = 2$) to retention test ($M = 65\%$, $SE = 2$).

To verify that the effect of order was consistent across both posttest and retention test, we conducted a separate Explanation Condition × Order ANCOVA for each test occasion. At both posttest and retention test, there was a main effect of order, $F(1,152) = 4.39$, $p = .038$, $\eta_p^2 = .03$, and $F(1,152) = 3.97$, $p = .048$, $\eta_p^2 = .03$, respectively. There was no effect of explanation condition or interaction between order and explanation condition at either test occasion (Fs < 1.3).

Intervention

As shown in the preceding analyses, children who solved problems prior to receiving instruction on the underlying concepts demonstrated greater conceptual knowledge on subsequent assessments. We next sought to better understand the reasons for this improved learning. Pretest knowledge and age were included as covariates in all analyses.

Midtest

Children completed a midtest between the problem-solving and instruction blocks of the intervention. Two of the midtest items were procedural knowledge items. A 2 (Order) × 2 (Explanation Condition) ANCOVA on accuracy on these items revealed no differences between conditions ($M = 67\%$, $SE = 3$, Fs < 1.64).

In contrast, there were differences between conditions in conceptual knowledge, but the effects varied with the item type. One midtest item asked children to define the equal sign. Children who had just received conceptual instruction on the meaning of the equal sign were more likely to give a relational definition of the equal sign (58% of children in the instruct–solve condition) than children who had only solved problems at that point (20% of children in the solve–instruct condition), $\chi^2(1, N = 159) = 23.18$, $p < .001$.

The other two conceptual knowledge items assessed children's encoding of the problem structures by asking children to reproduce equivalence problems from memory. Past work has demonstrated that children often make systematic errors when reconstructing problems (e.g., they reproduce $5 + 4 + 8 = 5 + \square$ as $5 + 4 + 8 + 5 = \square$; McNeil & Alibali, 2004). Children in the solve–instruct group were more likely to encode the structure of the problems correctly at midtest ($M = 54\%$ correct, $SE = 4$) than children in the instruct–solve group ($M = 44\%$, $SE = 4$), $F(1,152) = 4.16$, $p = .043$, $\eta_p^2 = .03$. This finding is striking given that children in the instruct–solve condition were explicitly taught about the structure of the problems and guided through several examples. Exploring the problems allowed children in the solve–instruct condition to better notice the structure of the equations than children who had received explicit instruction.

Problem-solving accuracy

We next examined children's accuracy on the problem-solving portion of the intervention. In all analyses with intervention problems, we report findings using the first set of intervention problems only (i.e., Problems 1–6) to equate the dependent variable for all groups. However, the same pattern of findings is obtained when including both sets of intervention problems for the extra practice groups.

Because children in the solve–instruct group solved problems as an exploratory tool (i.e., without prior instruction), we expected these children to solve problems less accurately than children in the instruct–solve group. A 2 (Order) × 2 (Explanation Condition) ANCOVA on intervention problem-solving accuracy confirmed this prediction. A main effect of order was found, $F(1,152) = 6.36$, $p = .013$, $\eta_p^2 = .04$. Children in the solve–instruct group ($M = 52\%$, $SE = 4$) were less accurate when solving the intervention problems compared with children in the instruct–solve group ($M = 65\%$, $SE = 4$). No other significant effects were found (Fs < 1).

Strategy use

We next examined the percentage of intervention problems on which children used each strategy type with separate 2 (Order) × 2 (Explanation Condition) MANCOVAs for correct and incorrect strategies. As shown in Table 3, children in the instruct–solve group used correct strategies more often, $F(5,148) = 3.24$, $p = .008$, $\eta_p^2 = .10$, and incorrect strategies less often, $F(4,149) = 3.07$, $p = .018$, $\eta_p^2 = .08$, than children in the solve–instruct group—consistent with their superior accuracy. However, children in the instruct–solve group still used incorrect strategies to solve more than a quarter of the problems.

Strategy variability

Given that children in the solve–instruct group were exploring the intervention problems prior to instruction, we expected these children to try a greater number of strategies during the intervention. For each child, we calculated the total number of different discernible strategies used on the intervention problems (correct strategies: equalizer, grouping, add–subtract; incorrect strategies: add all, add to equals; see Table 3) and submitted these totals to separate 2 (Order) × 2 (Explanation Condition) ANCOVAs for correct and incorrect strategies. No significant differences were found for the number of correct strategies used ($M = 1.26$, $SE = 0.06$, Fs < 2.13). Even though children in the solve–instruct

Table 3
Mean percentages correct and incorrect strategy use during tutoring intervention problem-solving block.

	Sample explanation: 3 + 4 + 8 = □ + 8	Instruct–solve	Solve–instruct
(A) Correct strategies			
Equalizer	3 + 4 is 7, 7 + 8 is 15, and 7 + 8 is also 15	51 (34)	36 (34)*
Grouping	I took out the 8s and I added 3 + 4	10 (17)	7 (16)
Add–subtract	I did 8 + 4 + 3 equals 15 and subtract 8	6 (11)	3 (9)
Incomplete procedure	I added 7 plus 8 (gave correct answer)	4 (12)	8 (11)[a]
Insufficient work	I used my fingers	4 (9)	2 (6)[a]
(B) Incorrect strategies			
Add all	I added 8 and 3 and 4 and 8 together	5 (10)	7 (13)
Add to equals	I just added 3 + 4 + 8	6 (14)	15 (21)*
Don't know	I don't know	5 (13)	5 (14)
Other incorrect	I just added 8 to 3	8 (15)	16 (23)*

Note: Standard deviations are in parentheses. "Other incorrect" strategies were counted as one category of incorrect strategies in analyses.
* $p < .05$.
[a] $p < .10$.

Table 4
Mean subjective ratings of understanding during tutoring intervention.

		Block 1	Block 2
Condition	Instruct–solve	1.68 (0.47) Instruct	1.68 (0.36) Solve
	Solve–instruct	1.53 (0.46) Solve	1.70 (0.38) Instruct

Note: Standard deviations are in parentheses.

group used correct strategies less often across all of the problems, these children did discover the same number of correct strategies as those in the instruct–solve group. For incorrect strategies, a main effect of order was obtained, $F(1,152) = 7.12$, $p = .008$, $\eta_p^2 = .05$. Children in the solve–instruct group used a greater number of different incorrect strategies ($M = 0.74$ out of 2, $SE = 0.07$) than children in the instruct–solve group ($M = 0.47$, $SE = 0.07$). No other effects were found (Fs < 1). Children in the solve–instruct group employed a greater number of incorrect strategies involving common misconceptions about the equal sign than children who had received instruction before problem solving.

Subjective ratings of understanding

The difficulty experienced during the initial exploratory problem-solving phase may have led the solve–instruct group to gain more accurate perceptions of their understanding (or lack thereof). This more accurate perception could, in turn, have alerted the solve–instruct group to the importance of the subsequent instruction. To explore this idea, we analyzed children's subjective ratings of understanding after each block of the intervention in a 2 (Order) × 2 (Explanation Condition) × 2 (Block: 1 or 2) ANCOVA. A significant Order × Block interaction was found, $F(1,152) = 5.81$, $p = .017$, $\eta_p^2 = .04$. No other condition effects reached significance (Fs < 2.83). As shown in Table 4, after Block 1, children in the solve–instruct condition rated their understanding as lower than children in the instruct–solve condition, $F(1,152) = 6.12$, $p < .05$, $\eta_p^2 = .04$. After Block 2, children in the solve–instruct condition rated their understanding as high as children in the instruct–solve condition ($F < 1$).

To gauge whether children in the solve–instruct group were more accurate in their assessments of understanding, we correlated final subjective ratings of understanding with conceptual knowledge at posttest and retention test. At posttest, ratings of understanding were significantly correlated with conceptual knowledge for both groups (solve–instruct: $r_p = .24$, $p = .039$; instruct–solve: $r_p = .23$, $p = .042$), indicating that both groups accurately rated their understanding immediately prior to completing the posttest. At retention test, the correlation between ratings of understanding and conceptual knowledge remained significant for the solve–instruct group ($r_p = .27$, $p = .020$). However, the relationship was no longer significant for the instruct–solve group ($r_p = .13$, $p = .249$). These findings suggest that children in the solve–instruct group more accurately perceived their understanding in a way that was maintained over time.

Explanation quality

Self-explanation prompts did not improve performance relative to solving extra practice problems. One reason may be the low frequency of conceptually oriented explanations. In both instructional orders, the frequency of explanations that included the concept of equivalence was relatively low (26 and 35% of explanations for the solve–instruct and instruct–solve groups that were prompted to self-explain, respectively, $F < 2$, *ns*). Rather, children's explanations of why an answer was correct or incorrect more often focused on reiterating the solution procedure.

Discussion

A key concern for parents, teachers, and researchers is how to structure learning situations to facilitate understanding (cf. Hirsch-Pasek et al., 2009; Piaget, 1973; Schwartz et al., 2007; Vygotsky, 1934/1987). In the current study, we examined how combining elements from two predominant theories of learning—exploration and explicit instruction—affects children's understanding of mathematical equivalence. Children solved relatively unfamiliar problems either before or after receiving brief instruction on the concept. Solving problems prior to receiving conceptual instruction (solve–instruct condition) resulted in significantly higher conceptual knowledge compared with a conventional instruct-then-practice approach (instruct–solve condition) at both an immediate posttest and a delayed retention test.

Incorporating an element of exploration in the learning environment had a marked benefit despite the fact that children in the solve–instruct condition used less sophisticated problem-solving strategies and showed poorer understanding during the tutoring intervention. The difficulty experienced by children in this condition appeared to help them more accurately assess their understanding, try a larger variety of incorrect strategies, and better encode the structure of the problems. Self-explanation prompts did not significantly improve children's understanding relative to additional practice. These findings document the importance of exploratory activities in combination with explicit instruction and reveal how certain learning activities can alter the ways in which children attend to the information they are learning.

Benefits of exploration

Although solving problems before instruction was more difficult than solving problems after instruction, there appear to have been several benefits to such struggle. Specifically, children may have been more likely to realize their limited understanding and attend to the learning activities at a deeper level.

Realizing limited understanding

Children who solved problems before instruction performed more poorly during the problem-solving portion of the tutoring intervention than children who received instruction first—solving problems less accurately, using poorer problem-solving strategies, and demonstrating lower explicit understanding of the equal sign midway through the intervention. These children, accordingly, rated their understanding as lower during the intervention compared with children in the instruct–solve condition.

Many researchers have noted the importance of struggle, cognitive conflict, relevant cognitive load, and/or difficulty during learning, arguing that a manageable level of such difficulty can ultimately lead to better learning and retention (Bjork, 1994; Eryilmaz, 2002; Hiebert & Grouws, 2007; Schmidt & Bjork, 1992; Sweller et al., 1998; Vamvakoussi & Vosniadou, 2004). The initial difficulties posed by exploratory practice in our study may have helped children to recognize their limited, and sometimes incorrect, prior knowledge and more accurately gauge their understanding of equations. Indeed, children's ratings of understanding during the intervention were correlated with their retention of conceptual knowledge over a 2-week delay in this condition. In contrast, in the instruct–solve condition, children's ratings of understanding were not correlated with their conceptual knowledge after a delay. Initial struggle may help children to better gauge their understanding.

Attending to learning activities at a deeper level

A better sense of limitations in one's knowledge may encourage children to more actively attend to the learning activities (Hiebert & Grouws, 2007). Children often fail to notice important information in the learning environment, and learning what information to attend to (i.e., encode) is a prominent process underlying learning and development (e.g., Case & Okamoto, 1996; Siegler, 1989). In the case of mathematical equivalence problems, children must notice that there are operations on both sides of the equal sign, but many children encode the problems as having operations only on the left side (e.g., reproduce $5 + 7 + 4 = 5 + \square$ from memory as $5 + 7 + 4 + 5 = \square$; McNeil & Alibali, 2004). Children in the solve–instruct condition encoded the structure of mathematical equivalence problems more accurately midway through the intervention than children in the instruct–solve group. This difference occurred despite the fact that the instruct–solve group had been explicitly taught the problem structure and is consistent with the idea that the lack of struggle can lead people to process information superficially (Bjork, 1994; Wittwer & Renkl, 2008). Exploring problems prior to instruction appears to have helped children to notice and encode important features of the problems, particularly the way in which the problem structure differs from problems they have commonly encountered in the past.

Children may have been more likely to notice these important problem features in the process of generating and revising potential solution strategies during exploration (Siegler, 1996). Children in the solve–instruct group used a larger variety of incorrect strategies than children in the instruct–solve condition, although they discovered correct strategies as well. This finding is particularly noteworthy because it contrasts a common notion that trying incorrect paths hurts performance. For example, associationist theories of learning and cognitive load theory (Kirschner et al., 2006; Seyler, Kirk, & Ashcraft, 2003; Siegler & Shipley, 1995) suggest that learning is advanced when the problem space is constrained as soon as possible so that erroneous associations are not reinforced. Such theories may underestimate the benefits of exploring a problem space. Generating and revising potential solution strategies may help children to identify and encode necessary features of the problems (Siegler, 1996).

Exploration and explicit instruction: joining forces

These findings have important implications for theories of learning and development that differ in their emphasis on the importance of exploration and explicit instruction from a social partner. In formal instructional contexts, several researchers have concluded that explicit instruction practices have received far more substantial empirical support (Alfieri et al., 2011; Kirschner et al., 2006; Mayer, 2004; Tobias, 2009). Although some note that forms of guided discovery are promising (e.g., Alfieri et al., 2011; Mayer, 2004), explicit instruction appears to be rising as the more strongly favored formal instructional approach in the research literature. It is also the dominant instructional model in U.S. mathematics education. For example, in representative eighth-grade lessons from six countries, at least two thirds of individual work time was spent solving problems using a teacher-demonstrated procedure, whereas less than a quarter of time was potentially spent developing new solution procedures or modifying previously learned procedures (with the least potential time spent being in U.S. lessons at 9%; Hiebert et al., 2003). The current study, drawing from constructivist theories, demonstrates the utility of adding exploratory activities to explicit instruction. It provides much-needed experimental evidence on the timing of explicit instruction (Mayer, 2009).

Attending to the ways in which explicit and exploratory experiences might work in combination more generally has important implications for theories of learning and development. For example, children can learn language both through explicit instruction from a social partner (e.g., pointing and labeling; Koenig & Harris, 2005; Tomasello et al., 2005) and through observations of language used around them (e.g., overhearing and associating words and their referents; Schneidman, Buresh, Shimpi, Knight-Schwarz, & Woodward, 2009; Smith & Yu, 2008). Perhaps children might more appropriately generalize from explicit labeling if they are first exposed to the word–referent pairing in their own explorations. The current findings suggest that other types of learning and development—such as language development—might be better understood by examining how, and in what order, explicit and exploratory experiences can be combined.

The role of self-explanation

Unlike exploratory practice, self-explanation prompts did not significantly improve performance. Self-explanation has been lauded as a powerful, domain-general learning process (e.g., Chi et al., 1994; Rittle-Johnson, 2006). However, a majority of the empirical support for self-explanation confounds self-explanation with time on task (i.e., children who are prompted to self-explain also spend more time on the task than children who are not prompted to self-explain). In the five published experimental studies in which time on task has been equivalent in the self-explain and no-self-explain conditions, two have found a benefit for self-explanation and three have not (Aleven & Koedinger, 2002; de Bruin, Rikers, & Schmidt, 2007; compared to Große & Renkl, 2006; Matthews & Rittle-Johnson, 2009; Mwangi & Sweller, 1998). For example, in Matthews and Rittle-Johnson (2009), all children received conceptual instruction and then solved problems, and self-explanation prompts did not improve learning compared with solving additional practice problems instead. We have replicated this finding and extended it to a context in which problem solving and self-explanation prompts occurred before instruction. It is important to note that only about a quarter of self-explanations contained conceptual ideas in the current study even though we attempted to support effective self-explanations by including easier problems thought to elicit early emerging understanding of the equal sign (e.g., $10 = 3 + \square$) and by including conceptual instruction before self-explanation for some children. These findings highlight difficulties in eliciting effective self-explanations.

Limitations and future directions

Despite the promising implications for theories of learning and instruction, these findings are based on a brief, one-on-one scripted instruction in a single, formal tutoring intervention. In addition, this intervention focused on just one fairly constrained activity that does not encompass the range of exploratory activities. For example, our exploratory activity included problems with which students were familiar as well as less familiar mathematical equivalence problems in an effort to activate relevant prior knowledge during exploration (cf. Carpenter et al., 2003; Dewey, 1910; Schwartz et al., 2007). Moreover, we demonstrated these effects in a domain where children often carry misconceptions into the learning situation. Although similar results have been shown in domains with less obvious misconceptions (e.g., cognitive psychology: Schwartz & Bransford, 1998; learning a new toy's function: Bonawitz et al., 2011), future work should examine whether there are characteristics of the learning domain that enhance or reduce the benefits of exploration prior to instruction. Future replications and extensions are needed before these methods can be generally prescribed to other contexts, such as the classroom and home, or to other exploratory activities, such as identifying causal factors through experimentation (e.g., Klahr & Nigram, 2004).

We also have demonstrated that, on average, exploratory activities lead to processes that improve learning. Future research should examine how individual differences also affect how children learn from exploratory activities and when these activities may be more or less important for understanding instruction (e.g., they may be less important when a person already has sufficiently differentiated prior knowledge; Schwartz et al., 2007).

Finally, this study has demonstrated several potential mechanisms for improved learning due to exploratory activities, and future work should examine these further. By better understanding the cognitive processes that underlie learning from exploration, we not only can better understand what factors improve learning but also can better predict the circumstances in which understanding will be best attained.

In conclusion, this study has demonstrated that exploratory activities can alter how children process new information, significantly improving subsequent conceptual understanding from instruction both immediately and over a 2-week time period. In addition to the implications for theories of learning and development, these findings have practical implications, demonstrating that exploratory activities do not need to be complex or time-consuming. Solving relatively unfamiliar problems with feedback can be sufficient to prepare children to learn from instruction.

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