Tuesday, June 18

History of Mathematics Education
- Past practices
- Key points in time
- TIMSS data

Teaching Students with Disabilities
- Needs vs. Reality
- Standards and Principles
- CCSSO Common Standards
- C-S-A intervention

Standards Based Lessons
- What are the characteristics?
- What do you need to learn?
- What knowledge and competence do you already have that is pertinent to what you want to learn?

Number one reason to understand the past – it informs the future

What has been tried?

How can we take the best from the 20th century and make a better 21st century?

TIMSS

- 1995 – Largest and most carefully designed study of mathematics performance and teachers' instruction
- More than 500,000 students (33,000 in US)
- Grades 4, 8 and 12
- 41 Nations
- Disturbing Trends

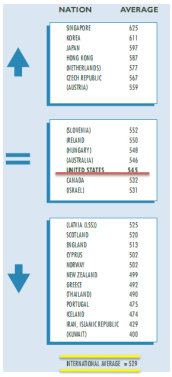
Fourth Grade

How did our US students do?

- ❑ Above International Average
- ❑ At the International Average
- ❑ Below the International Average

Grade 4 results

Fourth graders scored above the international average; they were below average in **measurement**, estimation and number sense!



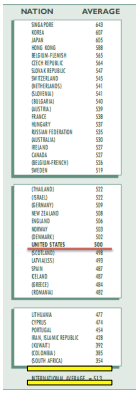
Eighth Grade

How did our US students do?

- ❑ Above International Average
- ❑ At the International Average
- ❑ Below the International Average

Grade 8 results

Eighth graders scored below the international average, outperformed by 20 countries. They were below the international average in geometry, **measurement** and proportionality.



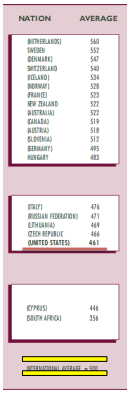
Twelfth Grade

How did our US students do?

- ❑ Above International Average
- ❑ At the International Average
- ❑ Below the International Average

Grade 12 results

US was only country in math to go from above international average at grade 4, to below international average at grade 8, to far below the international average at grade 12 !



Weakest Content Area in US

Which mathematics content was weakest?

- ❑ Number Sense
- ❑ Measurement
- ❑ Estimation
- ❑ Algebra
- ❑ Proportional Reasoning

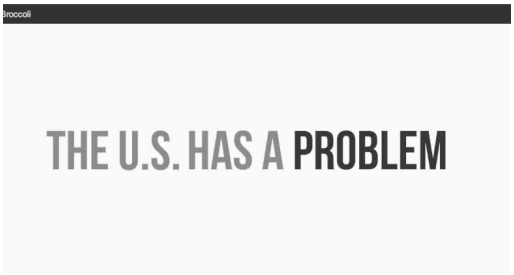
Why?

- We are only one of three countries NOT using the metric system (Liberia and Myanmar)
- We are only one of three countries NOT using the Celsius system (Belize and Jamaica)
- What??

Teaching Math in the US

- **Teaching results: (From a follow-up video study of lessons in grade 8 taught by US, German and Japanese teachers)**
  - US teachers teach students how to get answers while Japanese teachers teach for understanding

Video

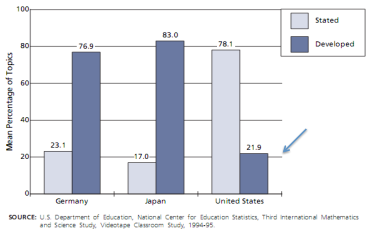


State or Develop

What percent of US lessons have math concepts that are developed rather than just stated?

- ❑ About 85%
- ❑ About 50%
- ❑ About 20%

Figure 18
Average percentage of topics in eighth-grade mathematics lessons that contained concepts that were stated or developed



SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study, Videotape Classroom Study, 1994-95.

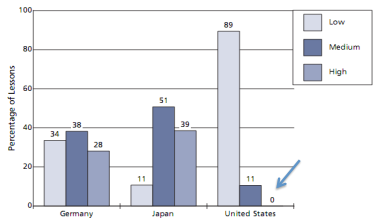
US teachers usually don't develop mathematical concepts/ideas

What is the Level of the Math Content

What percent of the mathematics content in US lessons was at a level of high quality?

- ❑ 75%
- ❑ 35%
- ❑ 0%

Figure 34
Percentage of lessons rated as having low, medium, and high quality of mathematical content



The United States teachers taught lessons rated as consisting of mostly low level mathematics content! None were rated as having high quality content!

NAEP – National Assessment of Education Progress

- Nation's Report Card
- Grades 4, 8 and 12
- Four Performance Levels
  - Below Basic
  - Basic
  - Proficient
  - Advanced

How do you think we're doing?

NAEP DATA

- What percent of fourth graders could correctly answer this question:
  - Which is larger ¼ or ⅓?

- ❑ 21%
- ❑ 37%
- ❑ 65%

Only 21%

- Unit fraction comparison task
- 41% of 8th graders could put three fractions in order from least to greatest
- You have to ask yourself:

How can students work with fractions (add, subtract, etc.) if they don't understand the magnitude of fractions?????

Do you teach fractions as a number – or do you start with ---This is the numerator – this is the denominator???

What did they say?

Students in eighth grade were asked to estimate (no paper and pencil allowed) 12/13 + 7/8. What do you think was the most common answer?

- ❑ 1
- ❑ 2
- ❑ 19
- ❑ 21
- ❑ Don't know

NAEP Results

Estimate the answer to 12/13 + 7/8

8th graders answered:

• 1	7%
• 2	24%
• 19	28%
• 21	27%
• Don't know	14%

I thought they had it!

The sum of ½ and ⅞ is closest to

A. 20
B. 8
C. ½
D. 1

Explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

Petit, M., Laird, R., & Marsden, E. (2010). *A focus on fractions: Bringing Research to the Classroom. New York:* Routledge. .

NAEP 2009

- Results from 2009 *National Assessment of Educational Progress* (NAEP)
  - 4th grade:
    - 42% regular education and 19% of students with disabilities at or above proficient.
    - 15% regular education and 41% of students with disabilities below basic (i.e., below grade level).
  - 8th grade:
    - 37% regular education and 9% of students with disabilities at or above proficient.
    - 23% regular education and 64% of students with disabilities below basic. (NCES, 2009)

How can we reach proficiency by 2014?

Why?

Awareness --- Concern ---- Change

Ramping up to Success

No "expiration" date rules

Teacher Knowledge Plays a Role

- ½ ÷ ¼ =

Liping Ma's Research

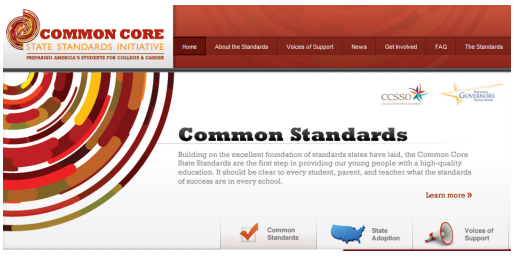
Students with Moderate/Severe Disabilities

- Shift from functional math to grade level expectations – core content
- Moved from using money to full range of content topics
- Stuck – with traditional math and functional approaches???
- Goal – Blend best practice for teaching students with disabilities and the research-based practice of the NCTM Standards

Shifts in Classroom Environment

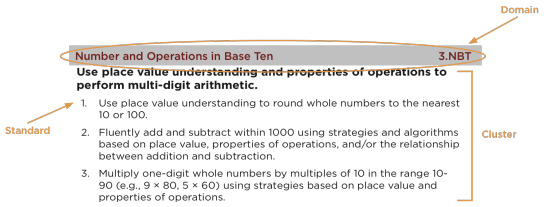
Away From:	Toward:
Classrooms as collections of individuals	Classrooms as mathematics communities
Teacher as "sole authority" for right answers	Logic and mathematical evidence as verification
Mere memorization of procedures	Mathematical reasoning
Emphasis on mechanistic finding of answers	Conjecturing, inventing, and problem solving
Treating mathematics as a body of isolated concepts and procedures	Connecting mathematics, its ideas, and applications

CCSS – www.corestandards.org



Design and Organization

- *Content standards* define what students should understand and be able to do
- *Clusters* are groups of related standards
- *Domains* are larger groups that progress across grades



Design and Organization

Grade Level Overviews

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CCSS – M CONTENT DOMAINS

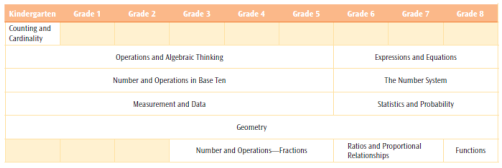


FIGURE 1.1 *Common Core State Standards* domains by grade level.

Standards of Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Rationale

- IDEA mandates that students with disabilities should have access to the new CCSS mathematics curriculum to the extent appropriate
- IDEA mandates that students with disabilities have individual educational plans that meet their specific needs based on their present level of performance
- How can both be accomplished?

What are the Issues?

- Not the problem: difficulty reading, paying attention, or following directions
- What is the problem? Underdeveloped cognitive structures which are the mental processes necessary to connect new information with prior knowledge.

Mathematically Literate Citizens

- Understanding big numbers – national debt - $13.8 trillion – How many years would it take to pay it back at $1 million a day?
- Lottery – Someone's got to win
- Mortgages -

KY Core Academic Standards

- Adopted June 2012
- ELA and Mathematics are the CCSS
- http://education.ky.gov/curriculum/docs/Documents/Kentucky%20Common%20Core%20MATHEMATICS.pdf

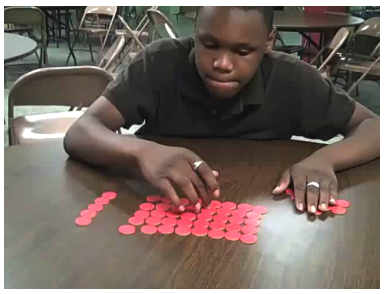
Kentucky Alternate Assessment Standards-3rd Grade

Alternate K-PREP Content Aligned Standards	KCAS Standard
(M-3.1) Interpret products of whole numbers.	**KCAS (3.OA.1)** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
(M-3.2) Demonstrate understanding of place value by rounding two digit whole numbers to the nearest 10.	**KCAS (3.NBT.1)** Use place value understanding to round whole numbers to the nearest 10 or 100
(M-3.3) Tell time to the nearest minute and measure time intervals by solving word problems.	**KCAS (3.MD.1)** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

So…What does reformed instruction look like?

- Hands on
- Respectful of prior knowledge
- Students communicating ideas
- Students assess their responses for reasonableness

Thinking about Ronny!!



Your Responsibilities

- What does it mean to DO mathematics?
- How does a person go about LEARNING mathematics?
- As a teacher of mathematics to students with disabilities, how should you go about teaching mathematics through problem solving?

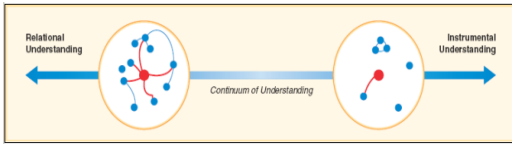
Reducing Resistance, Developing Responsibility and Building Resilience

- Give children choices and capitalize on their unique strengths
- Nurture traits of resilience
- Demonstrate an ethic of caring
- Make mathematics irresistible
- Give students some **leadership** in their own learning

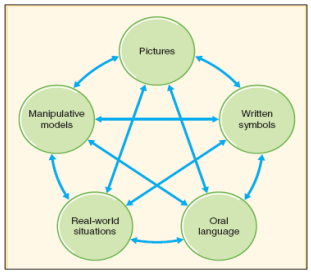
So, what do we need to do differently?

- **We need to help students develop understanding of mathematical concepts!**
- **We need to teach lessons which challenge students – lessons which involve high level tasks**

Their brains might look very different!



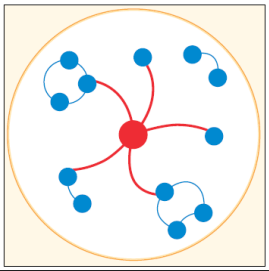
Five Representations of Mathematical Ideas



Why should we consider using multiple representations?

SO – one way of teaching for understanding is to help students connect new knowledge to existing knowledge!

That leads to a well-connected brain!



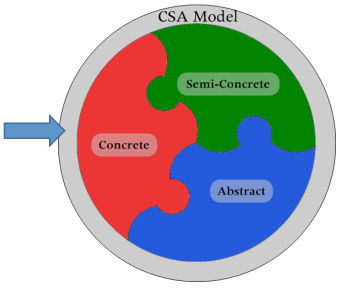
Implications

- Build new knowledge from prior knowledge
- Provide opportunities to talk about mathematics
- Build in opportunities for reflective thought
- Encourage multiple approaches
- Treat errors as opportunities for learning
- Scaffold new content
- Honor diversity

Implications for Teaching

- The need to replace the question "Does the student know it?" with the question "How does the student understand it?"
- Use of diagnostic interviews

CSA – Concrete Semi-Concrete Abstract



Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Recommendations for identifying and supporting students struggling in mathematics

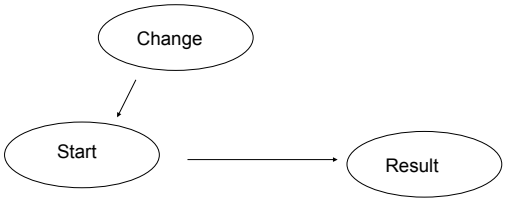
- Recommendations are based on strong and moderate levels of evidence resulting from comprehensive reviews of current research literature.
- Google IES practice guide

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC*: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies. ed.gov/ncee/wwc/publications/practiceguides/.

Warm up for today's topic

- I want you to use a number family like 9, 6, 15 to create a story problem for us to solve.
- Example- The donut tree grew donuts during the day and a donut burglar ate them at night. The tree had 15 donuts growing during the day, but in the morning only 6 were left. How many did the donut burglar eat?

Additive Structures



Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.
Carpenter, T., Fennema, E., Franke, M. L., Levi, L. & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Graphic Organizer (yes you need to use the beans/counters)

Join Problems: Three quantities

Louise has 11 baseball cards. Elliott gave her 6 more. How many baseball cards does Louise have now?

Louise has 11 baseball cards. Elliott gave her some more. Louise has 17 cards. How many did Elliott give her?

Louise has some baseball cards. Elliott gave her 6 more. Now she has 17. How many baseball cards did Louise have to begin with?

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.
Carpenter, T., Fennema, E., Franke, M. L., Levi, L. & Empson, S.. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Additive Structures – Separate Problems

Separate Problems: Initial is the whole or the largest amount (not the result)

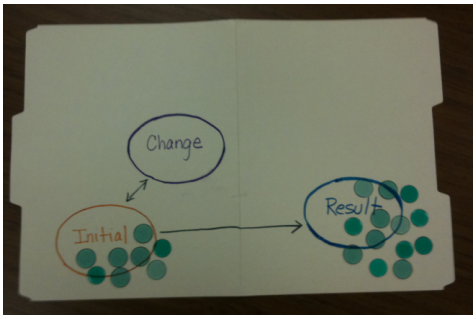
Mikal has 12 T-shirts. He gives 3 shirts to his brother Elron. How many T-shirts does Mikal have now?

Mikal has 12 T-shirts. He gives some shirts to his brother Elron. Now he has 9 left. How many T-shirts did Mikal give Elron?

Mikal has some T-shirts. He gives 3 shirts to his brother Elron. Now he has 9. How many T-shirts did Mikal have to begin with?

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.
Carpenter, T., Fennema, E., Franke, M. L., Levi, L. & Empson, S.. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Finding a Use for Old Folders



CCSS Appendix – Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

CCSS Appendix – Common Multiplication and Division Situations

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 × 6 = ?	3 × ? = 18, and 18 ÷ 3 = ?	? × 6 = 18, and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Two parts that are combined into one whole

Lynnette has 4 fiction and 3 nonfiction books. How many books does she have?

Lynnette and her friend Victoria put 7 books into a backpack. Lynnette put in 4 books. How many books did Victoria put in the backpack?

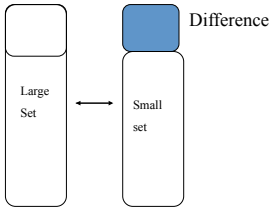
Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Where does this model lead?

- Y = B + Q
- Y = Q + B
- Y - Q = B
- Y - B = Q
- Y > B by Q

Dougherty, B. (2008). Measure Up: A Quantitative View of Early Algebra. In J. Kaput, D. Carraher & M. Blanton. *Algebra in the Early Grades*. Erlbaum.

Subtraction Structure



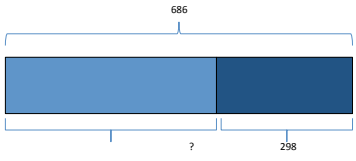
Compare Problems - involve two quantities, the third amount is not actually present but is the difference.

Ken has 12 pennies and Sue has eight. How many more does Ken have than Sue?

Ken has 12 pennies and Sue has eight. How many fewer does Sue have then Ken?

Singapore Strip Diagrams

- Mary made 686 biscuits. She sold some of them. If 298 were left over, how many biscuits did she sell?



686 – *x* = 298

Beckmann, S. (2004). Solving Algebra and Other Story Problems with Simple diagrams: *The Mathematics Educator*, 14(1), 42.-46.

Danger - Key Words Ahead

Mark has 3 packages of pencils. There are 6 pencils in each package. How many pencils does he have in all?

9

Explicit instruction - structure of problems

Why use these structures?

Key word strategies don't work

- no development of meaning making
- no building of structures for more advanced learning (decimals, fractions, mixed numbers)
- many problems do not have key words
- students use key words inappropriately
- multi-step problems are impossible to solve with key words

Students must learn to create **equivalent equations**: ☐ + 4 = 12 can be written equivalently as 12 – 4 = for the computational form

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Which would students say are true? False?

7 = 7

2 + 5 = 4 + 3

5 + 1 = 7

7 = 2 + 5

- Why?

Diagnostic Interview

- Emphasizes the collection of **in-depth** information about an individual student's knowledge and mental strategies.
- Provides evidence of **students' prior knowledge, naïve understandings** and **ways of thinking**
- Focuses on a task/problem where students are asked to verbalize their thinking and/or demonstrate ideas through **multiple representations**
- Is **not a teaching opportunity**
- Uses errors to **identify barriers to understanding** and to **inform instructional decisions**

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

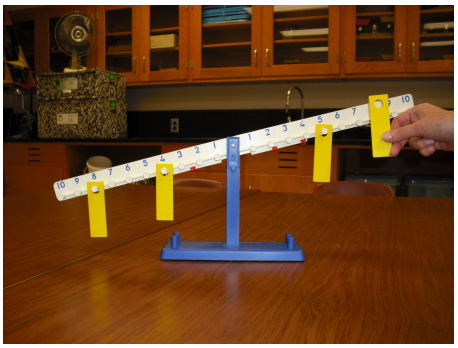
Diagnostic Interviews for Progress Monitoring

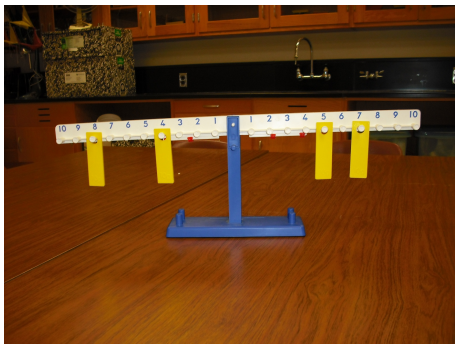
Give a task

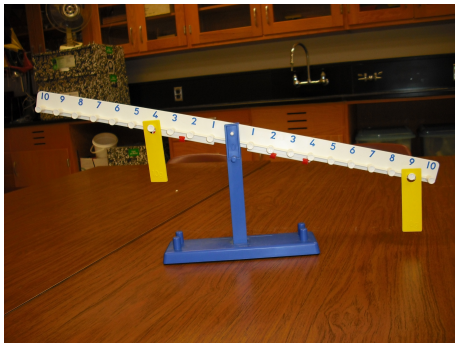
No instruction – just ask questions

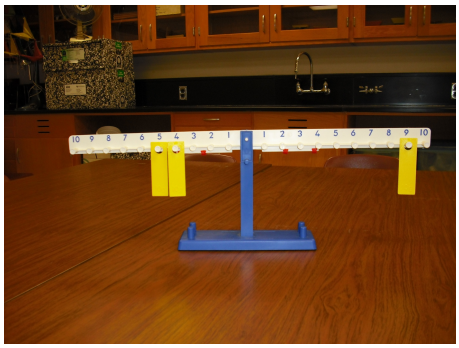
Capture student feedback on their thinking

$8 + 4 = \square + 5$









Talk to your Neighbor

How can you use the number balance to show these two situations?

- 3 X 4 = 12 ?
- $2x + 3 = 17$?

Equal Sign - Two Levels of Understanding

Operational - students mistakenly see the equal sign as something they must "do" with the numbers such as "give me the answer."

- Are students acquiring an appropriate understanding of the equal sign when you ask them to explain their thinking?
- See if they are comfortable using operations on both sides of the equal sign and can use the meaning of equal as "is the same as"

Relational - students can use the relationships between the two quantities to balance the sides of the equation. More advanced relational thinking will lead to the student generalizing rather than actually computing the individual amounts. They see the equal sign as one that relates to "greater than," "less than," and "not equal to."

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Common Core State Standards

Grade 1:
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

Relational Thinking Recap

Contextual - Set a problem situation – through a real world event or a story (children's literature) don't start with "naked" numbers

Concrete- e.g., part-whole models

Pictorial- ties into context and concrete models- so important to build that visualization of what is happening

Symbolic- (numeric/algebraic)

Verbal

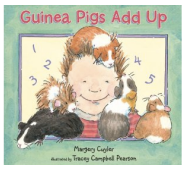
(create the idea) situate it, model it, draw it, say it, write it

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Guinea Pigs Add Up

K.OA.2:

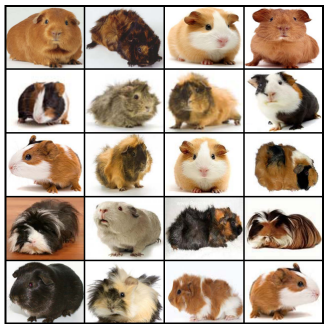
Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.



Sometimes Images work Better – *Guinea Pigs Add Up* - Cuyler

Guinea Pig Game Board

Number of Pigs	Number found homes	Number left



Both Addends Unknown

5 + 5 = 10
8 + 2 = 10
1 + 9 = 10
10 + 0 = 10
4 + 6 = 10
3 + 7 = 10
2 + 8 = 10
9 + 1 = 10
0 + 10 = 10
6 + 4 = 10
7 + 3 = 10

"Its a pattern. First, I put 5 and 5 in each cage and then 8 and 2, then 9 and 1, then 10 and 0. I then thought some were missing and figured them out."

Courtade, G., Lingo, A., Karp, K., & Whitney, T. (2013). *TEACHING Exceptional Children, 45*(3), 34-44.

USING SHARED STORY READING TO TEACH MATHEMATICS SKILLS TO ELEMENTARY STUDENTS WITH MODERATE AND SEVERE DISABILITIES

Why?

- Students learn mathematics through the use of contexts.
- A background situation or story allows learning concepts and skills through the building of meaning
- Starting a mathematics lesson with a problem in a context engages learners
  - Also positions them in the role of a problem solver.
- Common Core State Standards Mathematical Practice "Make sense of problems and persevere in solving them" (CCSSO, 2010 p. 6).

What is Shared Story Reading?

- Interactive read alouds, modeled reading, or story-based lessons
- Teacher reads, students ***strategically*** interact with book and teacher
- How is this different from a straight read aloud?

Shared Story Reading for Students with MSD

- Federal mandates
- IDEA, NCLB
  - Instruction and assessment based on alternate achievement of grade level standards
- Goal for teachers = developing standards-based lessons that continue to correspond to the diverse needs of individual students

Research

- Evidence based practice that has been used successfully to promote literacy skills for students with MSD (Hudson & Test, 2011)
- Benefits =
  - increased responding of students to the literature
  - increase in communication skills
  - promoting listening comprehension
    - (Browder, Mims, Spooner, Ahlgrim-Delzell, & Lee, 2008; Browder, Trela, & Jimenez, 2007; Hudson & Test, 2011; Mims, Hudson, & Browder, in press).

Connecting Shared Story Reading and Mathematics

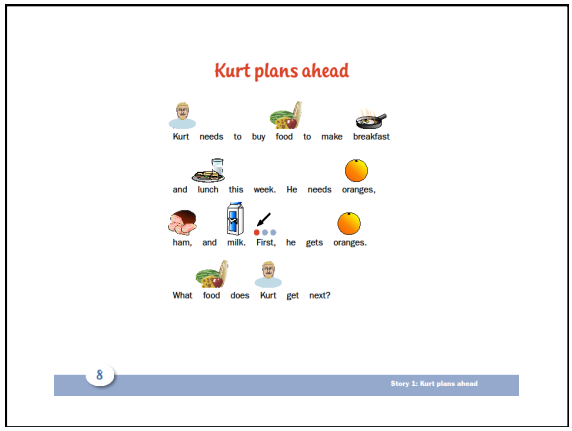
- Benefits should carry to other disciplines = MATHEMATICS!!
- "many children's books present interesting problems and illustrate how other children solve them. Through these books students see mathematics in a different context while they use reading as a form of communication" (National Council of Teachers of Mathematics, 1989, p. 27).

Connecting Shared Story Reading and Mathematics

- Incorporating SSR offers teachers the opportunity to
  - establish a different context
  - provide a schema to organize facts
  - provide a real-world connection to mathematical problems
    - (Anderson, Spiro, & Anderson, 1978; Zambo, 2005).
  - provides opportunities for students to engage in dialogue about mathematic
  - linking concepts to ideas in the world around them
    - (Pugalee, 2007; Young & Marroquin, 2006).

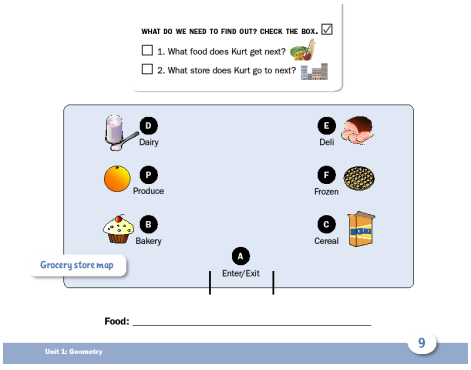
Existing Research

- Middle school-age students with MSD can successfully solve mathematics problems using a literature-based approach that embeds mathematics within a story context
  - (Browder, Trela, Courtade, Jimenez, Knight, & Flowers, 2012)
- Teaching to Standards: Math



Task Analysis for Geometry

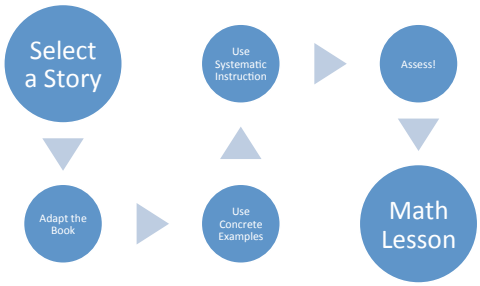
- Read story-based problem
- Identify problem statement
- Identify locations needed to solve problem (points, line segments, plane)
- Organize locations on graphic organizer
- Identify location to solve story problem
- Restate solution in story context



Template for Teaching to Math Standards

- Determine the Content with General Educators
- Task Analysis
  - Analyze the steps to solve the math problem
- Story
  - Write a story for the math problem
- Concrete Manipulatives
  - Create a "graphic organizer" and/or other manipulatives to learn the math operation
- Assistive Technology
  - Develop student response board or select AAC device to ensure student participation
- Teach and Monitor Progress
  - Teach the task analysis using systematic prompting and feedback until student masters

Template for Math Shared Stories



STEPS FOR USING SHARED STORY READING TO TEACH MATHEMATICS SKILLS

Step 1: Select a Book

- Book should relate to significant mathematical concept
  - Student's grade level expectations
  - Alternate achievement standards
  - IEP objectives

Step 1: Example 1

- **CCSSM-3.G.1**
  - Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- **KAAP (M-3.5)**
  - Understand that shapes in different categories may share attributes. Recognize rhombuses, rectangles and squares as examples of quadrilaterals.
- **Book Chosen:**
  - The Greedy Triangle by Marilyn Burns

Step 1: Example 2

- **CCSSM-3.OA.1**
  - Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
- **KAAP (M-3.1)**
  - Interpret products of whole numbers
- **Book Chosen**
  - Two of Everything by Lily Hong

Step 2: Adapt the Book

- Adaptations should meet the individual needs of students
  - Create a concept statement that will be repeated throughout the story
    - How to choose the concept statement
  - Choose and highlight important vocabulary
    - What is important?
  - Add visual/ kinesthetic cues that emphasize content to be learned

Step 2: The Greedy Triangle

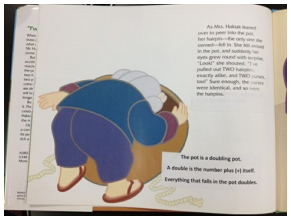
- Concept Statements:
  - *A quadrilateral has 4 sides. A quadrilateral has 4 angles.*
  - *A square is a quadrilateral.*
  - *A square has 4 sides that have the same length.*
  - *A square has 4 right angles*
- Vocabulary:
  - Triangle, Quadrilateral, Square, Rectangle, Rhombus, Sides, Angles

The Greedy Triangle

Step 2: Two of Everything

- Concept Statements:
  - *2 times a number is the number doubled.*
  - *A double is the number plus (+) itself*
- Vocabulary
  - Double, Doubling,
  - Multiply
  - Equal groups
  - Pattern
  - Skip count

Two of Everything



Step 3: Use Concrete Examples

- Models and manipulatives that represent the concepts
  - Remember CSA

Step 3

The Greedy Triangle

- Shapes on cards to identify
- Numbers on cards to choose the correct number of sides

Two of Everything

- Coins
- Doubling pot
- Numbers on cards

Step 4: Use Systematic Instruction

- Evidence-based practice for teaching academic skills to students with intellectual disability
- Constant Time Delay
  - To teach vocabulary, numerals
- System of Least Prompts
  - To teach mathematical concepts

CTD to Teach Vocabulary

- What is Constant Time Delay?
  - A response-prompting system that uses time increments to fade the prompt
  - In general, one response prompt is chosen (e.g., teacher modeling the target stimulus/ intended response)

SLP to Teach Concepts

- What is SLP?
  - A response-prompting system that uses a prompt hierarchy from least to most intrusive.
  - Hierarchy can include
    - verbal (telling the student what the expected response it)
    - model (pointing to or performing the expected response as a model for the student)
    - physical prompt (physically guiding the student to give or complete the response)

Step 5: Incorporate Assessments

- Need to monitor progress!
- When?
  - Before, during, and after shared story reading
- How?
  - Assess identification of numbers, shapes
  - Assess prompts needed to identify properties of two-dimensional shapes (sides, angles)

Assessing Geometry Vocabulary

Student: _____ Teacher:_____
Objective: _____
_____ second delay

Date/Session: _____		Date/Session: _____	
Vocabulary Word	Student response (+/ -)	Vocabulary Word	Student response (+/ -)
Triangle		Triangle	
Quadrilateral		Quadrilateral	
Square		Square	
Rectangle		Rectangle	
Rhombus		Rhombus	
Side		Side	
Angle		Angle	
Total Correct: _____ % Correct: _____		Total Correct: _____ % Correct: _____	

Student response: + = independent correct; - = incorrect, prompted, no answer

Assessing identification of properties of two dimensional shapes

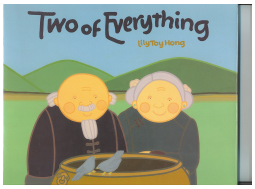
Student: _____ Teacher:_____
Objective: _____
Date/ Session:

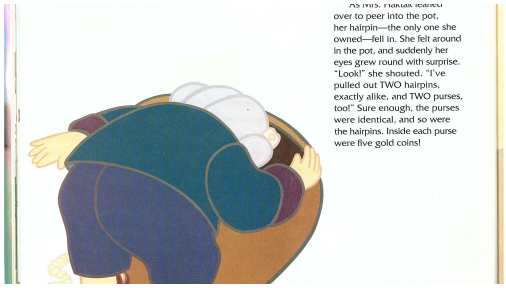
Shape	Student Response (I, V, M, P) Number of sides	Student Response (I, V, M, P) Number of angles
Triangle		
Quadrilateral		
Square		
Rectangle		
Rhombus		

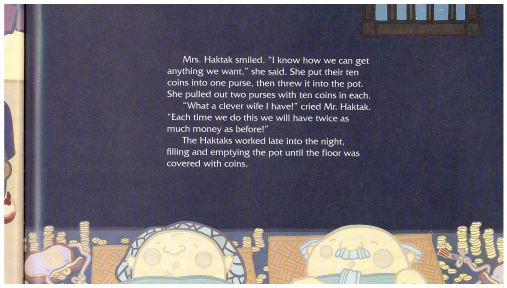
Total Independent Correct: _____
% Correct: _____

Student response: I= Independent correct, V= verbal prompt, M= model prompt, P= physical prompt

Two of Everything – Lily Toy Hong




As Mrs. Haktak leaned
over to peer into the pot,
her hairpin—the only one she
owned—fell in. She felt around
in the pot, and suddenly her
eyes grew round with surprise.
"Look!" she shouted. "I've
pulled out TWO hairpins,
exactly alike, and TWO purses,
too!" Sure enough, the purses
were identical, and so were
the hairpins. Inside each purse
were five gold coins!


Mrs. Haktak smiled. "I know how we can get
anything we want," she said. She put their ten
coins into one purse, then threw it into the pot.
She pulled out two purses with ten coins in each.
"What a clever wife I have!" cried Mr. Haktak.
"Each time we do this we will have twice as
much money as before!"
The Haktaks worked late into the night,
filling and emptying the pot until the floor was
covered with coins.

Functions – Finding the Rule

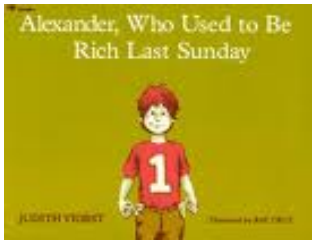
In	Out
1	2
2	4
3	6
4	8
5	
20	
n	

Understand that a function is a rule that assigns to each input exactly one output- enhancing algebraic thinking

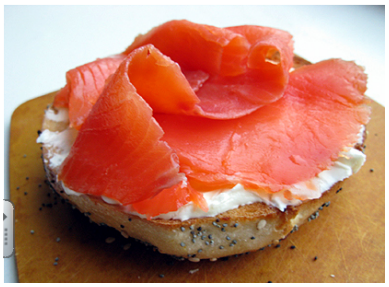
Using Children's Literature

- What is the book's birthday?
- *Alexander Who Used to be Rich Last Sunday*

Alexander



Lox



Stoop

Standards of Mathematical Practice

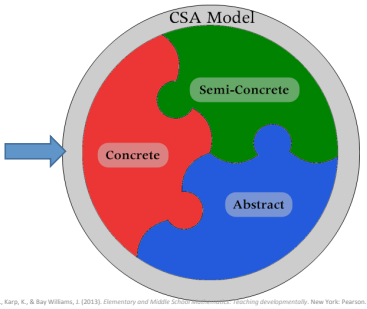
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Reducing Resistance, Developing Responsibility and Building Resilience

- Give students with disabilities **choices** and capitalize on their **unique strengths**
- Nurture traits of **resilience** – rather than learned helplessness
- Demonstrate an ethic of **caring**
- Make mathematics **irresistible**
- Give students with disabilities some **leadership** in their own learning

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally.* New York: Pearson.

Let's Recap



Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally.* New York: Pearson.

Equation	Word Problem
Model/Illustration	**Explanation**

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally.* New York: Pearson.

Algebraically	Graphically
Numerically in Tables	**Verbal Description**

Van de Walle, J., Karp, K., & Bay Williams, J. (2013). *Elementary and Middle School Mathematics: Teaching developmentally.* New York: Pearson.

Don't forget the importance of the Diagnostic Interview

- I realize how valuable a well-designed, research-based assessment can be in finding evidence of student understanding. Also how this awareness of children's thinking helped me decide what they (students) actually knew versus what I thought they knew.

A teacher from the Vermont Mathematics Partnership as quoted in Petit & Zawojewski, 2010

Petit, M. Zawojewski, J. (2010). Formative assessment in elementary school mathematics. In D. Lambdin & F. K Lester, Jr. (Eds.) *Teaching and learning mathematics: Translating research for elementary school teachers.* (pp. 73-79). Reston, VA: NCTM.

I thought they had it!

The sum of ½ and ⅞ is closest to

A. 20
B. 8
C. ½
D. 1

Explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

Petit, M., Laird, R., & Marsden, E. (2010). *A focus on fractions: Bringing Research to the Classroom.* New York: Routledge. .

Questions?

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