Preventive Maintenance and Replacement Scheduling:
Models and Algorithms

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TABELE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

Introduction

1.1. Preventive Maintenance and Replacement Scheduling

Preventive maintenance is a broad term that encompasses a set of activities aimed at improving the overall reliability and availability of a system. All types of systems, from conveyors to cars to overhead cranes, have prescribed maintenance schedules set forth by the manufacturer that aim to reduce the risk of system failure. Preventive maintenance activities generally consist of inspection, cleaning, lubrication, adjustment, alignment, and/or replacement of sub-components that wear-out. Regardless of the specific system in question, preventive maintenance activities can be categorized in one of two ways, component maintenance or component replacement. An example of component maintenance would be maintaining proper air pressure in the tires of an automobile. Note that this activity changes the aging characteristics of the tires and, if done correctly, ultimately decreases their rate of occurrence of failure. An example of component replacement would be simply replacing one or more of the tires with new ones.

Obviously, preventive maintenance involves a basic trade-off between the costs of conducting maintenance/replacement activities and the cost savings achieved by reducing the overall rate of occurrence of system failures. Designers of preventive maintenance schedules must weigh these individual costs in an attempt to minimize the overall cost of system operation. They may also be interested in maximizing the system reliability, subject to some sort of budget constraint. Other criteria such as availability and demand satisfaction might be considered as the objective functions, but they will not be studied in this dissertation. The problem is to find the best sequence of maintenance actions for each component in the system in each period over a planning horizon such that overall costs are minimized subject to a constraint on reliability or the reliability of the system is maximized subject to a constraint on budget.

1.2. Research Contributions

In this dissertation proposal, optimization models are developed and solved via exact, heuristic, and meta-heuristic algorithms. Analytical and statistical age-reduction and improvement factor models are developed and can be considered as the main research contribution. In particular, the following contributions are made:

1. Two optimization models will be constructed based on extensions of previous work in particular, by Usher *et al* (1998). The optimization models are solved by using a dynamic programming approach. These models also provide a general framework to achieve optimal preventive maintenance and replacement policies and, with modifications, can be used as a basic closed-form model for any type of system.
2. A multi-objective optimization model is developed based on a set of basic assumptions and engineering economy considerations. This model is optimized via multi-objective generational and steady state genetic algorithms as well as

by a multi-objective simulated annealing algorithm, which allows for the comparison of these optimization approaches.

3. In order to estimate the parameters of optimization models, an analytical model for estimating age reduction and improvement factor parameters will be developed. In addition, a procedure will be developed to estimate the improvement factor of any general component due to imperfect maintenance activities.
4. Finally, a real case study will be considered as the application of developed models and preventive maintenance and replacement schedule resulted from optimization models will be compared with the current maintenance policy in that case study.

1.3. Outline

The remainder of this dissertation proposal is organized as follows: In Chapter 2, a comprehensive literature review of models and applications of preventive maintenance and replacement scheduling is presented. In Chapter 3, a formulation of the optimization models is presented and their computational results are analyzed. Chapter 4 includes the extension of Chapter 3 optimization models by considering engineering economy features. These models have been optimized by multi-objective generational and steady state genetic algorithms and the computational results obtained by implementation of these algorithms are demonstrated. Finally, in Chapter 5, the plan and its schedule for the research is presented.

Chapter 2

Literature Review

2.1. Introduction

This chapter has four main sections. The first section presents a complete review on various optimization models and algorithms related to preventive maintenance and replacement scheduling. Section 2.3 presents a review of key works that utilize simulation models. In Section 2.4, models that introduce and develop age reduction and improvement factor models are presented. Finally, applications of preventive maintenance and replacement scheduling in manufacturing and production systems, service systems, and power systems are reviewed.

2.2. Optimization Models

2.2.1. Exact Algorithms

Deterministic optimization algorithms have been proposed by various authors. Yao *et al* (2001) present a two-layer hierarchical model that optimizes the preventive maintenance scheduling in semiconductor manufacturing operations. They develop a Markov decision process and optimize this model via a mixed integer linear programming model. They define profit of cluster tools production as the objective function to be maximized and consider time window for preventive maintenance activities and limitation of resources as the constraint, which were nonlinear functions. In order to achieve a global optimum, they transfer the nonlinear functions into linear ones and use EasyModeler and OSL as the optimization software. In addition, they utilize AutoSched AP as the simulation software in order to construct a simulation model to evaluate the performance of the optimization model in a real case study with 11 preventive maintenance tasks in a one-week planning horizon and compare the obtained results with the actual preventive maintenance plan. Later Yao *et al* (2004) extend their previous model to be more general, apply this extended model in a production line of a semiconductor manufacturing system, and show the application of it via numerical examples.

Jayakumar and Asgarpoor (2004) present a linear programming model in order to optimize the maintenance policy for a component with deterioration and random failure rate. They determine optimal mean times of minor and major preventive maintenance actions based on maximizing the availability of the component. They utilize MAPLE and LINGO for solving the linear programming model of Markov decision process. Duarte *et al* (2006) present a model and algorithm for maintenance optimization of a system with series components. In this research, they assume that all components have linearly increasing failure rate with a constant improvement factor for imperfect maintenance. In addition, they consider the total cost as the objective function and the total downtime as the main constraint. In terms of maintenance activities, they define preventive and corrective maintenance for each component. Finally, their algorithm optimizes the interval of time between maintenance actions for each component over a planning horizon.

Canto (2006) presents an optimization model to schedule a preventive maintenance of a real power plant over a planning horizon. He considers the total cost of various operations as the objective function and uses Bender's decomposition to solve a mixed-integer linear programming model. Budai *et al* (2006) present two mixed-integer linear programming models for preventive maintenance scheduling problems. The authors assume the total cost including possession costs, maintenance costs, and the penalty costs of early consecutive maintenance activities as the objective function for both models. They present and prove a theorem about the NP-hard structure of the preventive maintenance scheduling problem and use GAMS to implement the optimization models. They use CPLEX as the optimization software to find the optimal preventive maintenance schedule. They apply their model to a case study of railway maintenance scheduling. In addition, they develop four heuristic optimization algorithms, two for each model, and compare the computational results obtained from exact algorithms in CPLEX with the results achieved from heuristic algorithms and mention the advantages of each solution methodology.

Another excellent study in this area is by Tam *et al* (2006), who develop three nonlinear optimization models: one that minimizes total cost subject to satisfying a required reliability, one that maximizes reliability at a given budget, and one that minimizes the expected total cost including expected breakdown outages cost and maintenance cost. They utilize MS-Excel Solver as the optimization software that uses a generalized reduced gradient (GRG) algorithm to solve the nonlinear optimization models. Using these models, they determine the optimal maintenance intervals for a multi-component system but their models consider only maintenance actions for components and do not consider replacement actions. Alardhi *et al* (2007) present a binary integer linear programming model in order to find the best preventive maintenance schedule in separated and linked cogeneration plants. The researchers define the availability of the power and desalting equipments as the objective function to be maximized, and consider the maintenance time window, maintenance completion duration, logical operational, resource limitation, maintenance crew availability, efficiency measures, and demand as the set of constraints. They apply their model in two cogeneration plants with seven units and 42 pieces of equipment in Kuwait, over a 52-week planning horizon, and utilize LINGO as the optimization software to optimize the model. In addition, they perform a sensitivity analysis on the model to assess the robustness and analyze the effect of expanding the planning horizon, reducing the resources, and increasing the demand on the maintenance strategies.

Panagiotidou and Tagaras (2007) develop an optimization model that optimizes the preventive maintenance schedules in a manufacturing process. The authors consider two different states for components, in-control or out-of-control, before complete failure. They treat the time to shift and the time to failure as random variables and express them with Weibull and Gamma distributions. In addition, they combine age-based and condition-based concepts into the optimization model with the minimization of total cost and solve it by applying Karush-Kahn-Tucker (KKT) conditions of optimality to obtain the optimal preventive maintenance schedule. Finally, they present several numerical examples to demonstrate the effectiveness of their methodology. Shirmohammadi *et al* (2007) develop an age-based nonlinear optimization model to determine the optimal preventive maintenance schedule for a single component system. They define two types of decision variables, the time between preventive replacements and the cut-off age, and assume an expected cost of failures, maintenance, replacement costs, and total cycle cost in the cost function and consider cost per unit time as the objective function. In order to solve the optimization model and show the effectiveness of the proposed approach, they utilize MAPLE and run the program for a numerical example by setting different values for an improvement factor, which is assumed as a constant in the model.

Dynamic programming has been broadly used as a standard optimization technique to achieve the optimal maintenance and replacement actions in engineering problems. Canfield (1986) studies preventive maintenance optimization models via focusing on different aspects of failure function on systems reliability. He mentions that preventive maintenance actions do not change or affect deterioration behavior of failure rate, so the developed failure function is constant with maintenance actions. He considers increasing failure rate based on the Weibull distribution for his study and determines the optimal cost of maintenance policies by defining the average cost-rate of system operation and applying dynamic programming as the solution approach. Robelin and Madanat (2006) develop a maintenance optimization model for bridge decks via a Markov chain process. In this paper, they classify optimization models into two categories, (1) physically based deterioration models with limited number of decision variables, and (2) simpler deterioration models with more and sophisticated decision variables. They apply Markov chain methodology with states based on history of deterioration and maintenance actions and utilize dynamic programming as the solution approach to solve Markov decision process. As a case study, they apply their approach to optimize the maintenance policy of bridges.

2.2.2. Heuristics and Meta-Heuristics Algorithms

Genetic algorithm, as a major optimization approach, has been presented in several research papers. Usher *et al* (1998) present an optimization maintenance and replacement model for a single-component system. They determined an optimal preventive maintenance schedule for a new system subject to deterioration, by considering the time value of money in all future costs, increasing rate of occurrence of failure over time and the use of the improvement factor to provide for the case of imperfect maintenance actions. In addition, they provide a comparison of computational results among random search, genetic algorithm, and branch and bound algorithms.

One of the most notable studies in the area of reliability and maintenance optimization for multi-state multi-component systems is found in Levetin and Lisnianski (2000). They define a multi-state system as a system in which all or some of components have different performance levels, from proper functioning to complete failure and the reliability of the system as its ability of satisfying the demand levels. They formulate an optimization model to determine preventive maintenance actions that affect the effective age of components. Their model is based on minimization of cost subject to required level of reliability. They apply a universal generating function technique and use a genetic algorithm to determine the best maintenance strategy. Levetin and Lisnianski (2000) present additional research in which an optimization model was developed in order to determine the optimal replacement scheduling in multi-state series-parallel systems. They considered an increasing failure rate based on the expected number of failures during time intervals and defined summation of maintenance activities cost along with cost of unsupplied demand due to failures of components as the objective function. Finally, they utilized universal generating function approach and applied genetic algorithm to find the optimal maintenance policy.

Wang and Handschin (2000) develop a new genetic algorithm by modifying the basic operators, crossover and mutation, of a standard genetic algorithm based on the specific characteristic of preventive maintenance scheduling problem for power systems. They improve the time computational complexity of genetic algorithm by considering a code-specific and constraint-transparent integrated coding method to achieve faster convergence and to prevent production of infeasible solutions. As the implementation methodology, an object oriented programming approach is applied and the effectiveness of the new genetic algorithm shown via theoretical analysis and simulation results to compare with a traditional genetic algorithm. Tsai *et al* (2001) consider two activities, imperfect maintenance, and replacement, in their preventive maintenance optimization model. They model imperfect maintenance activities based on the concept of an improvement factor, which is determined by a quantitative assessment procedure. They use a genetic algorithm to find the optimal preventive maintenance activities while the system unit-cost life is considered as the objective function. As a case study, they test a mechatronic system to show the effectiveness of their model and algorithm.

Cavory *et al* (2001) present an optimization model to schedule the best preventive maintenance tasks of all machines in a single product manufacturing production line. They assume that each machine should be assigned to each operator and considered the total throughput of the line as the objective function to be maximized. At the first step, they formulate the optimization model and analyze it via analytical approach. Then, the researchers used C++ as a programming environment and applied genetic algorithm in order to find the best combination of preventive maintenance tasks. In addition, they construct an experimental design to set and analyze the parameters of genetic algorithm and utiliz the Taguchi method and statistical analysis to validate the results. Finally, an application of the approach was performed in an actual production line of car engines. Leou (2003) presents an optimization model to find the optimal preventive maintenance schedule for a multi-component system. He considers total cost of operations and maintenance activities along with reliability as the criteria of the system and transfer them into the objective function by defining degree of violation from required reliability. In addition, he defines maintenance crew and duration of maintenance as the system's constraints. He applies his optimization model in a case study with six electric generators and utilizes genetic algorithm as the optimization methodology to determine the best preventive maintenance schedule.

Han *et al* (2003) consider the recursive nature of failure rate between preventive maintenance cycles and develop a nonlinear optimization model based on repair cost, preventive maintenance cost, and production loss cost in a production system. They apply a genetic algorithm as the optimization technique and mention that their model can be considered in decision support systems for maintenance and job shop scheduling. Bris *et al* (2003) consider cost and availability as the systems criteria in their research. They optimize a model including cost in the objective function and availability as the constraint by using a genetic algorithm to find the best preventive maintenance schedule. They use a time-dependent Birnbaum importance factor to generate the ordered sequence of first inspection times and utilize MATLAB to calculate the system availability via a Monte Carlo simulation approach.

Limbourg and Kochs (2006) propose several techniques to represent the decision variables in preventive maintenance scheduling models that use heuristics and meta-heuristics optimization algorithms. They test various non-standard approaches and compare them to binary representations by a heuristic algorithm and the computational results show that effectiveness of their approaches. In addition, they apply some modified crossover and mutation procedures in a genetic algorithm and show the improvement in performance of their algorithm in terms of computational time and accuracy. Other research on the application of genetic algorithms to maintenance optimization has been recently done by Lapa *et al* (2006). They consider flexible intervals between maintenance actions and mention the advantage of this assumption over the common methodologies of continuous fitting of the schedules. They develop a model that includes preventive and corrective maintenance actions and the associated cost with them, outage times, reliability of the system, and probability of imperfect maintenance. Because their model is a non-linear large-scale optimization model, they utilize a genetic algorithm as the solution procedure. In addition and as a case study, they apply their model to a high-pressure injection system to measure the effectiveness of their methodology.

Verma and Ramesh (2007) group systems and sub-systems of a large engineering plant into higher modular assemblies (HMA) and apply a multi-objective preventive maintenance scheduling method. They model this problem as a constrained nonlinear multi-objective mathematical program with reliability, cost, and non-concurrence of maintenance periods and maintenance start time factor as elements of the objective functions and use a genetic algorithm to solve the model. Shum and Gong (2007) recently present an application of a genetic algorithm for optimization of preventive maintenance scheduling of a production machine. They consider maintenance and replacement frequency along with purchasing strategy and the size of the maintenance workforce as the decision variables and the total cost as the objective function. They examine the effect of these costs on the optimal maintenance schedule in numerical example.

Other meta-heuristics have been used as the combinatorial optimization techniques to solve maintenance scheduling problems. Samrout *et al* (2005) use an ant colony algorithm to optimize the problem that was previously optimized via genetic algorithm. They define series of component maintenance and inspection periods and use MATLAB as the programming environment.

2.2.3. Hybrid Algorithms

Kim *et al* (1994) combine genetic algorithm with simulated annealing in order to optimize a large-scale and long-term preventive maintenance and replacement scheduling problem. In their research, the acceptance probability of a simulated annealing method is considered as a measure for individual survival in the genetic algorithm. By using this approach, they achieve a near optimal solution in a short period of time compare to the computational time of simple genetic algorithm. As a case study, they optimize a long-term maintenance scheduling problem of a thermal system. Tan and Kramer (1997) develop a general framework for preventive maintenance optimization in chemical process operations. They assume a Weibull model for failure rate and consider different maintenance activities that can be performed. They develop a methodology that combines Monte Carlo simulation with a genetic algorithm to solve opportunistic maintenance problems with a non-deterministic objective function. They apply their approach to two case studies to compare the results obtained from the proposed model with the results achieved from analytic approach, and Monte Carlo simulation with a neural network. Finally, they mention the advantages of their approach over other approaches.

Marseguerra *et al* (2002) develop a condition-based maintenance (CBM) model for multi-component systems and use a Monte Carlo simulation model to predict the degradation level in a continuously monitored system. They apply a genetic algorithm to optimize the degradation level after maintenance actions in a multi-objective optimization model with profit and availability as the objective functions. In addition, they consider the simulation model to describe the dynamics of a stress-dependent degradation process in load-sharing components. Based on the computational results, they mention that the combination of a genetic algorithm with Monte Carlo simulation is an effective approach to solve the combinatorial optimization problems. Shalaby *et al* (2004) develop an optimization model for preventive maintenance scheduling of multi-component and multi-state systems. They define sequence of preventive maintenance activities as the decision variables and the summation of preventive maintenance, minimal repair, and downtime costs as the objective function. In addition, they consider system reliability, minimum intervals between maintenance actions, and crew availability as the constraints of their model. Finally, a combination of genetic algorithm and simulation was utilized to optimize the model.

Allaoui and Artiba (2004) present a combination of simulation and optimization models in order to solve the NP-hard hybrid flow shop scheduling problem with maintenance constraints and multiple objective functions based on flow time and due date. In addition, they consider setup times, cleaning times, and transportation times in the model and mention that the performance of the algorithm can be affected by the number of the breakdown times. Finally, they prove that the effectiveness of the simulated annealing algorithm is better than other heuristic algorithms with the same conditions.

Suresh and Kumarappan (2006) develop an optimization model and use a combination of genetic algorithm with simulated annealing. The authors apply their method to determine the preventive maintenance schedule in a power system. They mention that the method could produce better solutions if some changes and modification are made to the solution procedure. As a case study, they test the method on 62-unit state electrical system of Victoria. Samrout *et al* (2006) present another paper about the combination of an ant colony algorithm and genetic algorithm to optimize a large-scale preventive maintenance problem. They divide the objective function of their problem into two sections and then utilize each algorithm to improve the sections separately. They mention that using hybrid algorithm in a large-scale problem is more efficient than the simple algorithm.

2.2.4. Multi-Objective Algorithms

Multi-objective preventive maintenance optimization models have been presented in several papers. Kralj and Petrovic (1995) present a novel approach in preventive maintenance scheduling of thermal generating systems. The authors develop a large-scale multi-objective combinatorial optimization model with three objective functions and a set of the constraints. They consider minimization of total fuel costs, maximization of reliability in term of expected unserved energy, and minimization of technological concerns as the objective functions. In addition, they define maintenance duration, maintenance continuity, maintenance season, maintenance sequence of thermal units of the same class, limitation on simultaneous maintenance of thermal units, and limitation on total capacity on maintenance due to labor and resources as the constraints. They develop a multi-objective preventive maintenance scheduling software based on a multi-objective branch and bound algorithm implemented in FORTRAN. Finally, the researchers apply their methodology to a real system of 8 power plants with 21 thermal units with 11 maintenance classes over 31 weeks as the planning horizon.

Chareonsuk *et al* (1997) develop a multi-criteria preventive maintenance optimization model to find the optimal preventive maintenance intervals of components in a production system. In this study, the authors consider an age-based failure rate for components by fitting a Weibull distribution to the data and define expected total cost per unit time and the reliability of the production system as the main criteria. In following, they utilize a preference ranking organization method for enrichment evaluations (PROMETHEE) as the solution approach and define the alternative decisions as the preventive maintenance intervals. By using this approach, they can aggregate preferences of alternatives by combining the weighted values of the preference functions of the complete set of criteria. As a case study, they apply their methodology in a paper factory and used PROMCALC as the optimization software. Finally, they mention the advantage of their approach in which decision makers and managers can input various criteria into the model and do sensitivity analysis on the optimal solutions.

Konak *et al* (2006) present a comprehensive study on multi-objective genetic algorithms and their applications in reliability optimization problems. They review 55 research papers and demonstrate the recent techniques and methodologies. Quan *et al* (2007) develop a novel multi-objective genetic algorithm in order to optimize preventive maintenance schedule problems. They define the problem as a multi-objective optimization problem by considering the minimization of workforce idle time and the minimization of maintenance time and mention that there is a tradeoff between the objective functions. As the solution procedure, they use utility theory instead of dominance-based Pareto search to determine the non-inferior solutions and show the advantage of this method via numerical example. Taboada *et al* (2008) present a recent study in this area. They develop a multi-objective genetic algorithm in order to solve multi-state reliability design problems. The authors utilize the universal moment generating function to measure the reliability and availability criteria in the system. They applied their approach into two examples; the first one is a system of five units connected in series in which each component has two states, functioning properly, or failure and the second one is a system of three units connected in series. In this system, each component has multi states with different levels of performance, which range from maximum capacity to total failure. They utilized MATLAB as the programming environment, and shown the effectiveness of their approach in terms of computational times and obtained non-inferior solutions.

2.3. Simulation Models

2.3.1. Monte Carlo Simulation

Bottazi *et al* (1992) present the results of a systematic collection of actual failure times and preventive and corrective maintenance activities of 900 buses over a period of five years. They create an updatable database to estimate the failure distributions and to evaluate the influence of systematic preventive and corrective maintenance actions. They consider the total cost and availability as the objective functions, apply Monte Carlo simulation approach to evaluate and compare different maintenance policies, and present the computational results. Billinton and Pan (2000) develop a model, which is based on the use of Monte Carlo simulation, to determine the total failure frequency and the optimum maintenance interval for a parallel-redundant system. The authors present a modified distribution function assuming an exponential distribution for component useful life period and the Weibull distribution for the wear out period. The procedure includes construction of a mathematical model and definition of the stopping rule in simulation for a parallel-redundant system. They state that if the shape parameter β of the Weibull distribution increases, the optimum maintenance interval decreases. Finally, they show that a two-component parallel-redundant system is a basis structure in minimal cut set analysis that is used in evaluation of power systems reliability.

Zhou *et al* (2005) present an approach for sequential preventive maintenance scheduling based on the concept of age reduction due to imperfect maintenance actions. They consider an assumption for the time of imperfect maintenance actions based on required reliability of the system. They utilize a hybrid recursive method based on an assumed improvement factor and increasing failure rate and develop an optimization model with a maintenance cost rate in the life cycle of the system as the objective function. Finally, they apply Monte Carlo simulation and describe how their computational results can be used in decision support systems for maintenance scheduling. Marquez *et al* (2006) develop a simulation model to find the best preventive maintenance strategy in semiconductor manufacturing plants. The authors model the age of equipment, availability of equipment, maintenance activity backlog, and preventive maintenance policies and consider different wafer production scenarios in a Monte Carlo continuous time simulation model. They analyze and compare the different maintenance strategies on the status of manufacturing equipments and operating conditions of the wafer production flow. Furthermore, they describe how the combination of age and availability-based models increases the throughput and provides better results than the simple age-based models.

2.3.2. Discrete-Event and Continuous Simulation

Goel *et al* (1973) present a simulation model and develop a statistical analysis that considers three different types of preventive maintenance activities for components by defining stochastic and deterministic decision variables as well as unavailability and cost as the objectives. In addition, they make a 2-level sequential fractional factorial design in order to facilitate their simulation. By designing the simulation model based on experimental design approach, their model produces the preventive maintenance schedule for ground electronics systems. Burton *et al* (1989) develop a simulation model to evaluate the performance of a job shop. In this research, the effectiveness of the preventive maintenance scheduling under different conditions such as shop load, job sequencing rule, maintenance capacity, and strategy is determined and presented.

Krishnan (1992) develops a simulation model to determine the maintenance schedule for an automated production line in a steel rolling mill plant. He considers three different maintenance policies as opportunistic, failure, and block with the percent of availability as the objective function. He shows that the existing maintenance policy only includes the failure and block maintenance actions. By using the historical data of maintenance activities in the simulation model, the optimal preventive maintenance schedule is obtained in the form of checklist. Mathew and Rajendran (1993) present a simulation model in order to determine the frequency of the shutdown for periodic system overhaul, preventive and corrective maintenance, and inspections in a sugar manufacturing plant. They utilize a time-dependent simulation model to minimize the total cost including maintenance costs and downtime losses.

Paz *et al* (1994) develop a two-stage knowledge base for a maintenance supervisor assistant system. This knowledge base interacts with the maintenance manger on a periodic basis to select the proper preventive maintenance plan for the next period. The first stage deals with an object-oriented computer simulation model to monitor different preventive maintenance schedules that include preventive maintenance polices, staffing policies, downtime costs, simultaneous downtime practices, travel time impacts, and blocking situations as the systems specifications. In addition, they consider overall machine availability, critical machine availability, worker utilization, cost of the maintenance activities, and work order completion time as the systems criteria. At the second stage, they make a knowledge engineering environment to use the computational results obtained from a simulation model and send feedback to the first stage. Joe (1997) develops a simulation model in order to evaluate different preventive maintenance strategies for a fleet of vehicles of the St. Louis metropolitan police department. He utilizes GPSS as the simulation software, analyzes several policies to improve the effectiveness and efficiency of operations, and presents the best policy.

Savar (1997) develops a simulation model in order to investigate effect of different preventive maintenance strategies in a just-in-time production system. He constructs a simulation model on a 5-station production system and considers throughput rate, average equipment utilizations, and total work-in-process as the performance measures of the production system. After running the simulation model and analyzing the computational results, he mentions that preventive maintenance and corrective maintenance policies have a high impact on the performance measures of just-in-time production systems and by combining the maintenance activities and just-in-time operations one can improve the effectiveness of the this kind of systems. Mohamed-Salah *et al* (1999) develop a simulation model in order to achieve opportunistic maintenance strategies in a multi-component production line. The authors consider two different strategies and define total cost as the function of preventive and corrective maintenance activities as well as fixed cost due to any stop or failure in production line. The first strategy assumes that the maintenance activities are allowed on all non-failed components if the difference between the expected preventive time of non-failed components and the failure instant of failed components is less than certain value. The second one considers that the maintenance activities are allowed on all non-failed components if the difference between the expected preventive time of non-failed components and the preventive time or corrective instant of failed components is less than certain value. They utilize PROMODEL and describe that the total cost function has a unique optimum. Finally, they express that the optimal interval of maintenance for the strategies is 5.5 and 3.5 days, respectively.

Greasley (2000) presents a simulation model to find the optimal maintenance planning in train maintenance depot for an underground transportation facility in UK. He develops a simulation based on two different situations. The first situation assumes there is no random arrival and the second one considers random arrivals and investigates the effect of the arrival on service level performance measures. He utilizes ARENA as the simulation software and shows the effectiveness of the maintenance policies obtained by the simulation model. Chan (2001) presents a simulation model to analyze the effects of preventive maintenance policies on buffer size, inventory sorting rules, and process interruptions in a flow line of a push production system. He presents the performance of the production system under different operational conditions and preventive maintenance policies.

Duffuaa *et al* (2001) present a generic conceptual simulation model for maintenance systems. They define this simulation model by constructing seven modules including an input module, maintenance load module, planning and scheduling module, materials and spares module, tools and equipment module, quality module and finally, a performance measure module. The authors mention that this model could be used to develop a discrete event simulation models in one of the commercial simulation softwares. In addition, they suggest that by using this model one can evaluate the need for contract maintenance and effect of availability of spare parts on performance measures in the system. Devulapalli *et al* (2002) develop a simulation model in order to determine the best preventive maintenance policies for bridge management systems (BMS). They utilize STROBOSCOPE as the simulation software and examine the conditions of bridges under different strategies. They apply their model to a set of bridges in Virginia and argue that the model can be used to provide various maintenance policies for a bridge management system.

Alfares (2002) presents a simulation model to obtain the preventive maintenance schedule for components of a detergent-packing line and considers two different situations in his model. The first one assumes a constant time interval that is not affected by maintenance actions or unexpected failures. In the second situation, the time interval is affected and restarted by maintenance actions or unexpected failures. In order to minimize the total cost, he develops a simulation model to optimize the maintenance schedule of components for each situation. Houshyar *et al* (2003) present a simulation model to measure the impact of preventive maintenance scheduling on the production rate of a machine. They utilize PROMODEL to make the simulation model and consider two different scenarios for the simulation run. They use statistical analysis on the simulation outputs in order to determine the impact of recommended yearly preventive maintenance on the production throughput of the machine. Finally, they mention that the preventive maintenance policy does not affect the production rate but can reduce yearly maintenance costs of the system.

Han *et al* (2004) develop a finite time horizon model to achieve preventive maintenance scheduling of manufacturing equipment based on setback based residual factors and use simulation to solve the model. They mention the consistency of computational results and shown that simulation is a useful and effective method to solve such models. Jin *et al* (2006) develop a preventive maintenance optimization model for a multi-component production process. They define a combination of mechanical service, repair, and replacement activities for each component and use Markov decision process to present the transition function of probability for maintenance activities. In addition, they consider required reliability of the system as the constraint and total preventive maintenance cost as the objective function of the model. A simulation approach was utilized to find the optimal schedule as the solution procedure. The authors describe that considering the combination of preventive maintenance activities can reduce more cost in comparison with the situation that different activities are considered separately.

One of the most recent studies on application of simulation in preventive maintenance scheduling is presented by Hagmark and Virtanen (2007). They develop a simulation model to determine the level of reliability, availability and corrective and preventive maintenance at the early stage of design. Their method considers repair time delays and effect of preventive maintenance on the system's failure observed by condition monitoring and diagnostic resources. Yin *et al* (2007) recently propose a simulation model in order to analyze the dynamic structure of maintenance systems. The researchers consider various subsystems such as preventive maintenance subsystem, defects subsystem, condition-based subsystem, failure subsystem, corrective maintenance subsystem, and performance subsystem and utilized SIMULINK to build up the model. They analyze the structure of components and the relation of their constraints in a maintenance system and present the advantages of the model over classical stochastic process methods in a numerical example. In addition, they mention that obtained simulation results express the dynamic nature of maintenance systems.

2.4. Age Reduction and Improvement Factor Models

Nakagawa (1988) presents a basic and notable approach for models that utilize improvement factor. The work has been referenced by many researchers. He develops two analytical models in order to find the optimal preventive maintenance schedule based on an assumption of increasing failure rate over time. The first model, called a preventive maintenance hazard rate model, calculates the average failure cost of minimal repairs along with costs of preventive maintenance and replacement under the assumption that preventive maintenance actions reduce the next effective age to zero, the failure rate is assumed to increase with the increasing the frequency of preventive maintenance actions. Furthermore, this model assumes that maintenance activities take place at fixed intervals between each predetermined replacement. The second model, called an age reduction preventive maintenance model, considers the average failure cost of minimal repairs as well as costs of preventive maintenance and replacement by assuming the age reduction after each minimal repair. In order to find the optimal schedule, both models are optimized by calculus methods. He applies the models in a numerical example and describes that based on obtained computational results the second model is more practical than the first model.

Jayabalan and Chaudhuri (1992) propose another referenced work on age reduction and improvement factors models. They develop an optimization model and a branching algorithm that minimizes the total cost of preventive maintenance and replacement activities. They assume a constant improvement factor and define a required failure rate. In addition, they assume a zero failure cost and do not consider time value of money for future costs. Their algorithm determines the optimal schedule of maintenance actions before each replacement action in order to minimize the total cost in a planning horizon. They utilize FORTRAN to implement the algorithm and prove the effectiveness of the algorithm via several numerical examples.

Dedopoulos and Smeers (1998) develop a nonlinear optimization model to find the best preventive maintenance schedule by considering the degree of age reduction as the variable in the model. The researchers define improvement factor, time and duration of preventive maintenance activities as the decision variables, consider fixed cost and variable cost for maintenance actions, and define the variable cost as a function of the degree of age reduction, the duration of the action and the effective age of the component. Moreover, they present the failure rate in each period as a recursive function of age reduction from a previous period and consider the net profit as the objective function of the model. They implement the model in GAMS and use GAMS/MINOS optimization software. Finally, the effectiveness of the model is shown via three numerical examples. Martorell *et al* (1999) present an age-dependent preventive maintenance model based on the surveillance parameters, improvement factor, and environmental and operational conditions of the equipment in a nuclear power plant. They consider risk and cost as the criteria of the model based on the age of the system and made the sensitivity analysis to show the effect of the parameters on the preventive maintenance policies. They express that the results obtained from their model are different from those resulted from the models that do not consider the improvement factor and working conditions.

Lin *et al* (2001) combine the models were developed by Nakagawa (1988) and present hybrid models in which effects of each preventive maintenance action are considered by two aspects; one for its immediate effects and the other one for the lasting effects when the equipment is put to use again. The authors construct two models that reflect the concept of maintainable and non-maintainable failure modes. In the first model, they assume that preventive maintenance and replacement time are independent decision variables and consider the mean cost rate as the objective function to be minimized. In the second model, they assume that preventive maintenance activities are performed whenever the failure rate of the system exceeds the certain level and like the first model, the mean cost rate is considered as the objective function. Finally, they present numerical examples to show the application of the developed models and mention that for a system with a Weibull life distribution, optimal schedules can be achieved analytically, but for the general case, it cannot be solved by analytic methods. Xi *et al* (2005) develop a sequential preventive maintenance optimization model over a finite planning horizon. They define a recursive hybrid failure rate based on the improvement factor and increasing failure rate in order to estimate the systems reliability in each period of planning horizon. In addition, they consider the total cost of preventive maintenance activities and assume that mean cost in each period is a function of required reliability and the improvement factor. Finally, they utilize a simulation approach to optimize the model and mention that the computational results can be used in a maintenance decision support system of job shop scheduling.

Jaturonnatee *et al* (2006) develop an analytical model in order to find the optimal preventive maintenance schedule of leased equipment by minimizing a total cost function. They define maintenance actions as preventive and corrective, each with associated costs, and then consider the concept of reduction in failure intensity function along with penalty costs due to violation of leased contact issues. They present a numerical example for a system with Weibull failure rate, solve the model analytically, and examine the effect of penalty terms on the optimal preventive maintenance policies. Bartholomew-Biggs *et al* (2006) present several preventive maintenance scheduling models that consider the effect of imperfect maintenance on effective age of component. The researchers develop optimization models that minimize the total cost of preventive maintenance and replacement activities. In this study, they assume a known failure rate to express the expected failures as a function of age and consider age reduction in the effective age, based on the concept of an improvement factor. They develop a new mathematical programming formulation to achieve the optimal maintenance schedule and utilize automatic differentiation as the numerical approach, instead of analytical approach, to compute the gradients and hessians in the optimization procedure, which is the global minimization of non-smooth performance function. Finally, the effectiveness of the presented model and algorithm is shown in several numerical examples.

El-Ferik and Ben-Daya (2006) present an age-based hybrid model for imperfect preventive maintenance. The authors review different policies and the models developed by other researchers and propose a new sequential age-based analytical model. They assume that the imperfect preventive maintenance activities reduce the effective age of the system but increase the failure rate and presented mathematical formulations to determine the adjustment factors for both failure rate and age reduction coefficient. They construct an optimization model based on their analytical models, consider the minimization of the total cost as the objective function and solve the optimization model via a new heuristic algorithm for a numerical example. One of the recent works on methods for estimating age reduction factor is by Che-Hua (2007). In this research, he considers an optimal preventive maintenance for a deteriorating one-component system via minimizing the expected cost over a finite planning horizon. He develops a model for estimating improvement factor to measure the restoration of component under the minimal repair. The proposed improvement factor is a function of effective age of component, the number of preventive maintenance actions, and the cost ratio of each maintenance action to the replacement action. Finally, the researcher could obtain the optimal preventive maintenance schedule for a case study with the Weibull hazard function by applying a particle swarm optimization method.

Cheng *et al* (2007) present a paper about models for estimating the degradation rate of the age reduction factor. They present two optimization models, which minimize the cost subject to required reliability. The first model has a periodic preventive maintenance time interval for every replacement and the second one contains the maintenance schedule where the time interval between the final maintenance and replacement is not constant. Lim and Park (2007) present three analytical preventive maintenance models that consider the expected cost rate per unit time as the objective function. In this research, they assume that each preventive maintenance activity reduces the starting effective age but does not change the failure rate and consider the improvement factor as the function of number of preventive maintenance activities. They also assume that the failure function is based on a Weibull distribution and develop mathematical formulation for three different situations; preventive maintenance period is known, number of preventive maintenance is known, and number and period of preventive maintenance is unknown. They derive the optimal preventive maintenance and replacement schedules by taking an analytical approach and apply them to a numerical example to show an application of their models.

2.5. Applications

2.5.1. Manufacturing and Production Systems

The application of preventive maintenance scheduling has been widely used in manufacturing and production systems. For example, Hsu (1991) develops an optimization model in order to determine the optimal preventive maintenance schedules for a serial multi-station manufacturing system. He mentions that most of models use simulation at that time but his model is focused on mathematical programming approach. The computational results of his study show that operating features of the stations are interrelated and one must investigate the effect of preventive maintenance activities on all stations at the same time. Cassady *et al* (1999) develop an integrated control chart and preventive maintenance scheduling to reduce the total operating cost of manufacturing systems. The researchers formulate an economic model that includes the product inspection costs, process downtime costs and poor quality costs and analyze it via a simulation model. In addition, they construct a simulation-optimization model in order to evaluate and optimize the parameters of control chart and preventive maintenance strategy. They demonstrate their approach in a numerical example and shown the feasibility and effectiveness of their methodology.

Westman and Hanson (2000) develop a model to determine the mean time to failure (MTTF) as a function of the uptime for a workstation in a multi-stage manufacturing system. The authors assume that the uptime of the workstation has an increasing rate and is reduced if preventive maintenance actions are performed. They mention that this methodology captures the flexibility and multi-stage properties of manufacturing systems and can generate the preventive maintenance policies. Westman *et al* (2001) formulate a mathematical model to find the optimal production scheduling via linear quadratic Gaussian Poisson function with state dependent Poisson process. They consider the total cost of production and maintenance policies as the objective function and demonstrate the application of the model by a numerical example.

Charles *et al* (2003) present a preventive maintenance optimization model in order to minimize the total maintenance costs in a production system. In this paper, they consider the total productive maintenance, corrective maintenance and preventive maintenance actions along with production operations as well as the related associated costs. They assume a Weibull life distribution and utilize MELISSA C++ as discrete-event production-oriented simulation software to evaluate different scenarios. As a case study, they analyze a prototype semiconductor manufacturing workshop to demonstrate the approach and mentioned that this model has general structure that can be applied for other kind of manufacturing systems. Han *et al* (2004) develop a nonlinear optimization model to minimize the cost of maintenance and replacement actions under the reliability constraints for production machine in a production system. Their model considers Weibull distribution as the failure function of the machine and can be used as a decision support system for job shop scheduling.

Sawhney *et al* (2004) present a simulation model to determine maintenance strategies of a manufacturing system. Their model is constructed for integrating reactive and proactive maintenance scheduling in order to increase productivity of operations in the lean manufacturing structure. Preventive maintenance optimization is also used in semiconductor manufacturing. Li and Qian (2005) present a real time preventive maintenance optimization model for cluster tools in a semiconductor manufacturing system. They consider the standpoint of the system and used genetic algorithm as the solution procedure.

In the area of application of preventive maintenance in manufacturing and production systems, many researchers are interested in integrating preventive maintenance and production scheduling. Adzakpa *et al* (2004) present an application of combination of maintenance scheduling and job assignment in distribution systems. They develop an optimization model that considers the total cost of maintenance actions as the objective function and availability in a given time-window and precedence among consecutive standby jobs and their emergency as the constraints of the model. They show that their problem is NP-hard to solve and because of that, they use a heuristic optimization algorithm to solve the problem. Ying *et al* (2005) develop an integrated model that simultaneously considers preventive maintenance and production scheduling decision variables. Their model minimizes the total tardiness of jobs and makes a 30% reduction in expected total tardiness of jobs.

Rezg *et al* (2004) present an integrated preventive maintenance and inventory control simulation model for a production line with multi-component. The authors define preventive and corrective maintenance activities along with inventory control variables and parameters to develop approximate analytical models for the single machine under different scenarios. In addition, they utilize PROMODEL to construct an age-based simulation model and apply genetic algorithm to optimize the parameters of the simulation model and evaluate different production scenarios. Finally, they test their methodology on three numerical examples of a production line and compare the computational results with results obtained from analytical formulas. They mention that applying combination of maintenance strategies production planning policies leads to a significant reduction of the total cost. Rezg *et al* (2005) present another paper in this area. He and his colleagues develop an integrated age-based preventive maintenance and inventory control simulation model in a manufacturing system with just-in-time configuration. They present two approaches; the first one is a mathematical model to determine the average cost per unit time and the second one is the combination of simulation and experimental design. They use MAPLE for solving the analytical model, utilize PROMODEL for simulation, and use STATGRAPHICS to analyze the data for experimental design and regression analysis. The authors mention that both approaches could give approximately same results and existing difference due to approximation assumptions considered in the analytical model that was eliminated in the simulation model.

Sortrakul *et al* (2005) present an optimization model of integrated preventive maintenance planning and production scheduling for a single machine. The authors mention that these problems have been tackled separately in several papers but they have not been considered together in real manufacturing systems. They consider the total weighted expected job completion time as the objective function and optimize the combinatorial optimization model via genetic algorithm. As the result, they express the advantages and effectiveness of their approach that can be used to solve real manufacturing problems. Cassady and Kutanoglu (2005) develop and present an integrated preventive maintenance and production scheduling mathematical model for a single-machine. They consider the total weighted expected completion time as the objective function to be minimized. Their model allows multiple maintenance activities and explicitly captures the risk of not performing maintenance. They use a heuristic approach to solve the model and compare the obtained computational results of integrated model with the results achieved from the solving preventive maintenance and job scheduling problems independently.

Leng *et al* (2006) present an integrated preventive maintenance scheduling and production planning multi-objective optimization model for a single machine. They use chaotic particle swarm optimization algorithm to solve the model and show the application and effectiveness via numerical examples. Li and Zuo (2007) recently develop a simulation model that determines that impact of preventive and corrective maintenance activities on the total cost of inventories in a production system. They apply simulation as the solution methodology to find the optimal number of failures and the optimal level of safety stock simultaneously and mention that combining the preventive and corrective maintenance with production scheduling can reduce the large amount of total operating cost in system.

Kou and Chang (2007) develop an integrated production and maintenance optimization model for a single machine based on cumulative damage process, and the effect of preventive maintenance policy on the production scheduling in order to minimization of the total tardiness. The authors express that in the optimal strategy if jobs have certain process time with different due dates, the optimal production schedule sorts the jobs by earliest due date and if jobs have certain due dates with different process time, it sorts them by shortest process time. In addition, they mention that the optimal maintenance policy is a constraint on the production schedule when machine shuts down due to cumulative damage failure process. The computational results show that by increasing the number of jobs the effect of jobs due dates on the optimal maintenance policy control limit is decreased. Zhou *et al* (2007) demonstrate an age based preventive maintenance scheduling combined with production planning optimization model in order to maximize the availability of a production machine. The authors use a heuristic algorithm to obtain the optimal schedule that minimizes the make span. They also apply a simulation approach to validate the heuristic algorithm and to show its effectiveness to solve the flow shop scheduling problems of integrated production and preventive maintenance.

Ruiz *et al* (2007) present comprehensive research in area of integrating preventive maintenance and production scheduling. They define three different policies for preventive maintenance scheduling; preventive maintenance at fixed predefined time intervals, preventive maintenance for maximizing the equipment availability, and maintaining a minimum reliability threshold over the planning horizon. The minimization of the total manufacturing time of the sequence is considered as the main criterion. The authors apply six different adaptations of heuristic and meta-heuristic algorithms to evaluate the last two policies for two sets of problems and mention that ant colony and genetic algorithm solve these problems effectively. Finally, they conclude that integrated preventive maintenance and production scheduling optimization problems along with meta-heuristic algorithms can be successfully applied in flowshop problems. In addition, they suggest that one can define more criteria and consider the problem as a multi-objective optimization model.

2.5.2. Service Systems

Jayabalan and Chaudhuri (1992) present two different preventive maintenance models for maintaining bus engines in a public transit network based on minimization of the total cost over a finite planning horizon. They construct the models based on the concept of mean time to failure (MTTF) of the engines and assume the upper bound for the failure rates. The first model is based on different Weibull failure functions between preventive maintenance activities and the second assumes that the each preventive maintenance action reduces the effective age of the system. The authors present the obtained computational results and show the effectiveness of the models in a real case study.

Pongpech *et al* (2006) present an optimization model that minimizes the total maintenance costs and penalty costs for used equipment under lease. They assume

Weibull distribution as the failure function for equipment, develop a 4-parameter model and apply a 4-stage algorithm to solve it. They apply their model to several numerical examples with different contract assumption and analyze the optimal policy in each situation.

Martin (1988) presents a preventive maintenance optimization model, which has been developed, and implemented by Columbia Hospital in Milwaukee based on plant technology and safety management standards. The hospital designed this program in order to use the optimal preventive maintenance plan for electrical distribution equipment with considering safety, serviceability, reliability and the total cost.

Fard and Nukala (2004) study and review the application of different stochastic process such as homogenous Poisson process (HPP), non-homogenous Poisson process (NHPP), branching Poisson process (BPP), and superimposed renewal process (SRP) in preventive maintenance scheduling. They present current methods based on non-homogenous processes for modeling and optimization of single and multi-component systems. They assume that maintenance actions do not affect the failure rate of system; hence, the NHPP can be applied to present and model repairable service systems.

2.5.3. Power Systems

Applications of preventive maintenance scheduling are not restricted to manufacturing or service systems. Power plants use preventive maintenance strategies to increase the reliability and availability of equipments. McClymonds and Winge (1987) present methods to achieve optimal preventive maintenance scheduling for nuclear power plants, though they have not been applied successfully. They consider the plant availability and reliability as the objective functions and develop models based on assigning resources to preventive and corrective maintenance activities. Zhao *et al* (2005) present an age-based preventive maintenance optimization model for a gas turbine power plant. They develop a model with profit instead of cost as the objective function and considered power plant performance, reliability and the market dynamics effects in the model. In order to determine the effects of economics on maintenance costs and frequencies, they utilize a sequential approach and show its effectiveness by using real data of based load combined cycle power plant with a gas turbine unit.

2.6. Chapter Summary

In this chapter, recent work pertaining to methods and applications of preventive maintenance and replacement scheduling were reviewed. They were categorized as optimization models, simulation models, age reduction and improvement factor models, and applications in manufacturing, service and power systems. We find that most studies focus on single-component systems or simple and specific systems, which is not always applicable for real and general systems. In addition, not much work has been done in the area of age reduction and improvement factor models. Hence, we propose preventive maintenance and replacement scheduling models that deal with multi-component system and can be applied to a wide variety of systems. Because we use the concept of age reduction and improvement factor in these models, we also develop mathematical and statistical models to estimate the improvement factor for imperfect maintenance activities. These are our research contributions and they are applied to a real system.

Chapter 3

Optimization Models - Exact Algorithms

3.1. Introduction

This chapter will present a new modeling approach to find optimal preventive maintenance and replacement schedules for multi-component systems. We construct new closed-form optimization models based on the cost and reliability characteristics and solve them using a standard optimization procedure. These models provide a general framework that can be applied and used in a wide variety of systems. Computational results show the feasibility of the proposed approach.

3.2. Formulation

Consider a new repairable series system of N components, each subject to deterioration. Each component i is assumed to have an increasing rate of occurrence of failure (ROCOF), $v_i(t)$, where t denotes actual time, $(t > 0)$. In this paper, we assume that component failures follow the well-known Non-Homogeneous Poisson Process (NHPP), with ROCOF given as:

$$v_i(t) = \lambda_i \beta_i t^{\beta_i - 1} \quad \text{for } i = 1, \ldots, N \tag{3.1}$$

where λ_i and β_i are the characteristic life (scale) and the shape parameters of component i respectively. The NHPP is similar to the Homogeneous Poisson Process

(HPP) with the exception that the failure rate is a function of time. For more on this well-known stochastic process see, Ascher and Feingold (1984).

We seek to establish a schedule of future maintenance and replacement actions for each component over the period [0, *T*]. The interval [0, *T*] is segmented into *J* discrete intervals, each of length *T/J*. At the end of period *j*, the system is either, maintained, replaced, or no action is taken. We assume that maintenance or replacement activities in period *j* reduce the "effective age" of the system and thus it is ROCOF. For simplicity we also assume that these activities are instantaneous, i.e., the time required to replace or maintain is negligible, relative to the size of the interval, and thus is assumed to be zero, however, we do impose a cost associated with the repair or maintenance action.

To account for the instantaneous changes in system age and system failure rate, we introduce the following notation. Let $X_{i,j}$ denote the effective age of component i at the start of period *j*, and $X'_{i,j}$ denotes the age of component i at the end of period *j*. It is clear that:

$$X'_{i,j} = X_{i,j} + \frac{T}{J} \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{3.2}$$

3.2.1. Maintenance

Consider the case where component i is maintained in period *j*. For simplicity, we assume that the maintenance activity occurs at the end of the period. The maintenance action effectively reduces the age of component i for the start of the next period. That is:

$$X_{i,j+1} = \alpha \cdot X'_{i,j} \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \text{ and } (0 \le \alpha \le 1) \tag{3.3}$$

The term α is an "improvement factor", similar to that proposed by Malik (1979) and Jayabalan and Chaudhuri (1992). This factor allows for a variable effect of maintenance on the aging of a system. When $\alpha = 0$, the effect of maintenance is to return the system to a state of "good-as-new". When $\alpha = 1$, maintenance has no effect, and the system remains in a state of "bad-as-old".

Note that the maintenance action at the end of period j results in an instantaneous drop in the ROCOF of component i, as shown in Figure 3.1. Thus at the end of period j, the ROCOF for component i is $v_i(X'_{i,j})$. At the start of period $j+1$ we find that the ROCOF drops to $v_i(X'_{i,j})$.

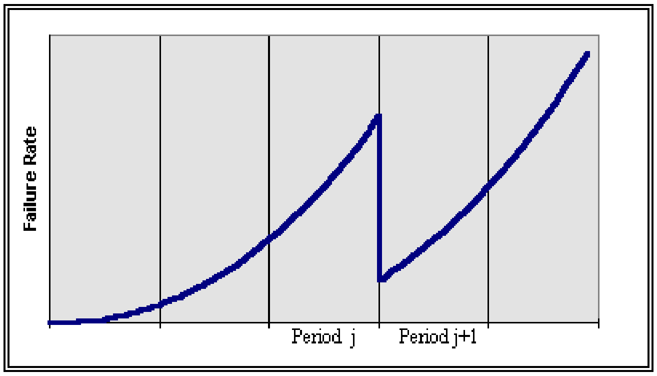

Figure 3.1. Effect of period-*j* maintenance on component ROCOF

3.2.2. Replacement

If component i is replaced at the end of period *j*, we find that:

$$X_{i,j+1} = 0 \quad \text{for} \quad i = 1,\dots,N;\ j = 1,\dots,T \tag{3.4}$$

i.e., the system is returned to a state of "good-as-new". The ROCOF of component i instantaneously drops from $v_i(X'_j)$ to $v_i(0)$ as shown in Figure 3.2.

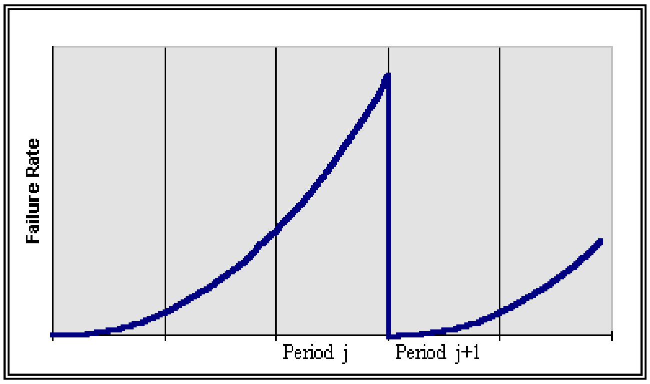

Figure 3.2. Effect of period-*j* replacement on system ROCOF

3.2.3. Do Nothing

If no action is performed in period *j*, we see no effect on the ROCOF of component *i*, and we find that:

$$X'_{i,j} = X_{i,j} + \frac{T}{J} \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{3.5}$$

$$X_{i,j+1} = X'_{i,j} \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{3.6}$$

$$v_i(X_{i,j+1}) = v_i(X'_{i,j}) \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{3.7}$$

3.2.4. Cost of Preventive Maintenance and Replacements

For a new system, we seek to find the cost associated with a given schedule of future maintenance and replacement activities. The cost associated with all component-level maintenance and replacement actions in period *j*, will be a function of the all the actions taken during that period.

3.2.4.1. Failure Cost

When we view the future periods of operation for the system, we must account for the inevitable costs due to unplanned component failures. From our vantage point, at the start of period $j = 1$, however, we cannot know when such failures will occure. However, we know that if the system carries a high ROCOF through a period, then we are at risk of experiencing high number, and hence, high cost of failures. Conversely, a low ROCOF in period j should yield a low cost of failure. To account for this, we propose the computation of the expected number of failures in each period for each component in the system. (We depart here from the approach found in Usher et al (1998) where an average failure rate concept was used with a cost constant.) Here we compute the expected number of failures of component i in period j, as:

$$E\left[N_{i,j}\right] = \int_{X_{i,j}}^{X'_{i,j}} v_i(t)\,dt \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{3.8}$$

Under the NHPP assumption, we find the expected number of component i failures in period j to be:

$$E\left[N_{i,j}\right] = \lambda_i\left(X'_{i,j}\right)^{\beta_i} - \lambda_i\left(X_{i,j}\right)^{\beta_i} \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{3.9}$$

We assume that the cost of each failure is F_i (in units of \$/failure event), which in turn allows us to compute, $F_{i,j}$ the cost of failures attributable to component i in period j as:

$$F_{i,j} = F_i\, E\left[N_{i,j}\right] \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{3.10}$$

Hence regardless of any maintenance or replacement actions (which are assumed to occur at the end of the period) in period j, there is still a cost associated with the possible failures that can occur during the period.

3.2.4.2. Maintenance Cost

If maintenance is performed on component i in period j, a maintenance cost constant M_i is incurred at the end of the period.

3.2.4.3. Replacement Cost

If component i is replaced, in period j, we assume that the replacement cost is the initial purchase price of the component i, denoted R_i.

3.2.4.4. Fixed Cost

For a multi-component system, and the cost structure defined above, the problem can be shown to reduce to a simple problem of finding the optimal sequence of maintenance, replacement, or do-nothing actions for each component, independent of all other components. That is, one could simply find the best sequence of actions for component 1 regardless of the actions taken to component 2 and so on. This would result in N independent optimization problems. Such a model seems unrealistic, as there should be some overall system cost penalty when an action is taken on any component in the system. It would seem that there should be some logical advantage to combining maintenance and replacement actions, e.g., while the system is shut down to replace one component, it may make sense to go ahead and perform maintenance/replacement of some other component, even if it is not at its individual optimum point where maintenance or replacement would ordinarily be performed.

Under this scenario, the optimal time to perform maintenance/replacement actions on individual components is dependent upon the decision made for other components. As such, we propose that a fixed cost of "downtime", Z, be charged in period j if any component (one or more) is maintained or replaced in that period. Consideration of this fixed cost makes the problem much more interesting, and more difficult to solve, as the optimal sequence of actions must be determined simultaneously for all components.

3.2.4.5. Total Cost

From our vantage point at the start of period $j = 0$, we wish to determine the set of activities, i.e., maintenance, replacement, or do nothing, for each component in each period such that total cost is minimized. In order to have $X'_{i,j}$, age of component i at the end of period j by using equation (3.2) first, we define $m_{i,j}$ and $r_{i,j}$ as binary variables of maintenance and replacement actions for component i in period j as:

$$m_{i,j} = \begin{cases} 1 & \text{if component } i \text{ at period } j \text{ is maintained} \\ 0 & \text{otherwise} \end{cases} \tag{3.11}$$

$$r_{i,j} = \begin{cases} 1 & \text{if component } i \text{ at period } j \text{ is replaced} \\ 0 & \text{otherwise} \end{cases} \tag{3.12}$$

Then, we construct the following recursive function of $X_{i,j}, X'_{i,j}, m_{i,j}, r_{i,j}, \alpha$ with a constraint:

$$\begin{cases} X_{i,j} = (1 - m_{i,j-1})(1 - r_{i,j-1})X'_{i,j-1} + m_{i,j-1}(\alpha . X'_{i,j-1}) \\ X'_{i,j} = X_{i,j} + \dfrac{T}{J} \end{cases} \tag{3.13}$$

$$m_{i,j} + r_{i,j} \leq 1 \tag{3.14}$$

In addition, we assumed the initial age for each component is equal to zero:

$$X_{i,1} = 0 \quad \text{for } i = 1, \ldots, N \tag{3.15}$$

If component replacement occurs in the previous period then $r_{i,j-1} = 1,\ m_{i,j-1} = 0$, so $X_{i,j} = 0$. If a component is maintained in the previous period then $r_{i,j-1} = 0,\ m_{i,j-1} = 1$ so $X_{i,j} = \alpha . X'_{i,j-1}$ and finally if we do nothing, $r_{i,j-1} = 0,\ m_{i,j-1} = 0$, and $X_{i,j} = X'_{i,j-1}$ which corresponds to our basic assumptions given in Section 3.1.

From our definitions of each type of cost, we can write the total cost function as:

$$Total\ Cost = \sum_{i=1}^{N}\sum_{j=1}^{T}\left[F_i . \lambda_i \left(\left(X'_{i,j}\right)^{\beta_i} - \left(X_{i,j}\right)^{\beta_i}\right) + M_i . m_{i,j} + R_i . r_{i,j}\right] + \sum_{j=1}^{T}\left[Z\left(1 - \prod_{i=1}^{N}\left(1 - \left(m_{i,j} + r_{i,j}\right)\right)\right)\right] \tag{3.16}$$

This objective function computes the total cost as a simple sum of component costs in each period based on any maintenance or repair cost, the system “downtime” cost, and the cost of the expected number of failures. It is certainly possible to compute a more accurate economic measure of these costs, such as Net Present Value (NPV), using a suitable interest rate. One could also include the effects of inflation, by adding an inflation rate in the calculation of future costs. While these may make the model more accurate, we have avoided those minor refinements for the sake of notational simplicity. See Usher et al (1998) for more on these issues.

3.3. Optimization Models

3.3.1. Model 1 - Minimizing total cost subject to a reliability constraint

In this model, we attempt to minimize the total cost subject to the constraint that some minimum level of system reliability over the planning horizon is achieved and assume that components are arranged in series. It is important to note that other system configurations (parallel, k-out-of-n, complex, etc) can be handled just by modifying and adapting the fixed cost section and the reliability function based on the configuration of the parallel, k-out-of-n, or other complex systems, but for the sake of simplicity, we consider only series systems in this paper.

To consider the reliability constraint in this model, we define the reliability function for component i in the period j as follow:

$$R_{ij} = e^{-E\left[N_{ij}\right]} \tag{3.17}$$

By this definition, we find the series system reliability to be:

$$R = \prod_{i=1}^{N} \prod_{j=1}^{T} e^{-E\left[N_{ij}\right]} \tag{3.18}$$

Then we formulate the following non-linear mixed integer-programming model that minimizes the total cost for a given reliability in the system:

$$Min\ \ Total\ Cost = \sum_{i=1}^{N}\sum_{j=1}^{T}\left[F_i\,.\lambda_i\left(\left(X'_{i,j}\right)^{\beta_i} - \left(X_{i,j}\right)^{\beta_i}\right) + M_i\,.m_{i,j} + R_i\,.r_{i,j}\right] + \sum_{j=1}^{T}\left[Z\left(1 - \prod_{i=1}^{N}\left(1 - \left(m_{i,j} + r_{i,j}\right)\right)\right)\right]$$

$s.t.:$

$$\begin{array}{ll}
X_{i,1} = 0 & i = 1,\dots,N \\
X_{i,j} = (1 - m_{i,j-1})(1 - r_{i,j-1})X'_{i,j-1} + m_{i,j-1}(\alpha.X'_{i,j-1}) & i = 1,\dots,N \text{ and } j = 2,\dots,T \\
X'_{i,j} = X_{i,j} + \dfrac{T}{J} & i = 1,\dots,N \text{ and } j = 1,\dots,T \\
m_{i,j} + r_{i,j} \le 1 & i = 1,\dots,N \text{ and } j = 1,\dots,T \\
\prod_{i=1}^{N}\prod_{j=1}^{T} e^{-\left[\lambda_i\left(\left(X'_{i,j}\right)^{\beta_i} - \left(X_{i,j}\right)^{\beta_i}\right)\right]} \ge R_{series} & \\
m_{i,j}, r_{i,j} = 0 \text{ or } 1 & i = 1,\dots,N \text{ and } j = 1,\dots,T \\
X_{i,j}, X'_{i,j} \ge 0 & i = 1,\dots,N \text{ and } j = 1,\dots,T
\end{array}$$

(3.19)

3.3.2. Model 2 - Maximizing reliability subject to a budget constraint

Here we modify the formulation and introduce a budget constraint, B. The objective of this model is to maximize the system reliability, through our choice of maintenance and replace decisions, such that we do not exceed the budgeted total cost. This model can be formulated as:

$$Max\ Reliability = \prod_{i=1}^{N}\prod_{j=1}^{T} e^{-\left[\lambda_i\left(\left(X'_{i,j}\right)^{\beta_i} - \left(X_{i,j}\right)^{\beta_i}\right)\right]}$$

$s.t.:$

$$\begin{array}{ll}
X_{i,1} = 0 & i = 1,\dots,N \\
X_{i,j} = (1 - m_{i,j-1})(1 - r_{i,j-1})X'_{i,j-1} + m_{i,j-1}(\alpha.X'_{i,j-1}) & i = 1,\dots,N \text{ and } j = 2,\dots,T \\
X'_{i,j} = X_{i,j} + \dfrac{T}{J} & i = 1,\dots,N \text{ and } j = 1,\dots,T \\
m_{i,j} + r_{i,j} \le 1 & i = 1,\dots,N \text{ and } j = 1,\dots,T \\
\sum_{i=1}^{N}\sum_{j=1}^{T}\left[F_i\,.\lambda_i\left(\left(X'_{i,j}\right)^{\beta_i} - \left(X_{i,j}\right)^{\beta_i}\right) + M_i\,.m_{i,j} + R_i\,.r_{i,j}\right] + \sum_{j=1}^{T}\left[Z\left(1 - \prod_{i=1}^{N}\left(1 - \left(m_{i,j} + r_{i,j}\right)\right)\right)\right] \le B & \\
m_{i,j}, r_{i,j} = 0 \text{ or } 1 & i = 1,\dots,N \text{ and } j = 1,\dots,T \\
X_{i,j}, X'_{i,j} \ge 0 & i = 1,\dots,N \text{ and } j = 1,\dots,T
\end{array}$$

(3.20)

3.4. Solution Procedure

Based on sequential nature of the preventive maintenance and replacement scheduling and nonlinear mixed-integer programming structure of the models presented in Section 3.2, we apply a Dynamic Programming (DP) approach to solve the models. Dynamic programming was developed and introduced by Bellman (1957) as a computational methodology for solving certain types of optimization problems. In this approach, an optimization problem involving n decision variables is reduced and transformed to a set of n single-variable optimization problem. Dynamic programming provides an excellent procedure to obtain an optimal solution when a problem involves one constraint, at most. However, the computational complexity of the approach increases exponentially with the number of constraints.

We defined state vectors x and x', to be the effective ages at the start and end of periods, in spaces X and X' called the state spaces, and decision vectors m and r, binary variables of maintenance and replacement actions, in spaces M and R called the policy spaces. Note that each policy in these spaces is a concatenation of elementary decisions and based on equations (13), (14), and (15) it can be presented as transition functions as follow:

$$\begin{cases} x_{i,j} = g_{i,1}(x'_{i,j-1}, m_{i,j-1}, r_{i,j-1}) & \text{for } i = 1,\dots,N;\ j = 1,\dots,T \\ x'_{i,j} = g_{i,2}(x_{i,j}) & \text{for } i = 1,\dots,N;\ j = 1,\dots,T \end{cases} \tag{3.21}$$

Suppose that the aim is to find the sequence of decisions that makes extreme value for a return function. We define $C_{1,i,j}$ as a separable function of failure, maintenance, and replacement costs for component i in period j, $C_{2,j}$ as a separable function of fixed cost in period *j*, and $R_{i,j}$ as separable function of reliability for component *i* in period *j*.

$$\begin{cases} C_{1,i,j}(x_{i,j}, x'_{i,j}, m_{i,j}, r_{i,j}) = F_i.\lambda_i\left(\left(x'_{i,j}\right)^{\beta_i} - \left(x_{i,j}\right)^{\beta_i}\right) + M_i.m_{i,j} + R_i.r_{i,j} \\ C_{2,j}(m_{i,j}, r_{i,j}) = Z\left(1 - \prod_{i=1}^{N}\left(1 - \left(m_{i,j} + r_{i,j}\right)\right)\right) \\ R_{i,j}(x_{i,j}, x'_{i,j}) = e^{-\left[\lambda_i\left(\left(x'_{i,j}\right)^{\beta_i} - \left(x_{i,j}\right)^{\beta_i}\right)\right]} \\ \text{for } i = 1,\dots,N;\ j = 1,\dots,T \end{cases} \tag{3.22}$$

We can transfer the nonlinear mixed-integer optimization models to dynamic programming models by applying the separable functions presented in equation (3.22), considering the first set of constraints as the initial conditions needed for recursion, and using the second and third sets as the transition functions presented in equation (3.21). Now, we define $f_k(w)$ and $f'_k(w')$ as recursive sequence of functional equations at stage *k* for the total cost and the reliability in which w and w' are the parameters that discretize the right hand side values of the main constraint on a certain grid. Finally, the dynamic programming formulation of optimization models can be presented as follows:

$$f_k(w) = \underset{\substack{m_{i,j};r_{i,j} \\ m_{i,j}+r_{i,j}\le 1}}{Min}\left\{\begin{array}{l} f_{k-1}\big((\prod_{j=1}^{k}\prod_{i=1}^{N} R_{i,j}(g_{i,1}(g_{i,2}(x_{i,j-1}), m_{i,j-1}, r_{i,j-1}), x'_{i,j})) - w\big) + \\ \left(\sum_{j=1}^{k}\left(\sum_{i=1}^{N} C_{1,i,j}(g_{i,1}(g_{i,2}(x_{i,j-1}), m_{i,j-1}, r_{i,j-1}), x'_{i,j}, m_{i,j}, r_{i,j})\right) + C_{2,j}(m_{i,j}, r_{i,j})\right) \end{array}\right\}$$

$$\text{for } k = 1,\dots,T;\ w = 1, 0.9, 0.8, \dots, R_{series} \tag{3.23}$$

$$f'_k(w') = \underset{\substack{m_{i,j};r_{i,j} \\ m_{i,j}+r_{i,j}\le 1}}{Max}\left\{\begin{array}{l} f'_{k-1}\Big(w' - \big(\sum_{j=1}^{k}\left(\sum_{i=1}^{N} C_{1,i,j}(g_{i,1}(g_{i,2}(x_{i,j-1}), m_{i,j-1}, r_{i,j-1}), x'_{i,j}, m_{i,j}, r_{i,j})\right) + C_{2,j}(m_{i,j}, r_{i,j})\big)\Big) \times \\ \left(\prod_{j=1}^{k}\prod_{i=1}^{N} R_{i,j}(g_{i,1}(g_{i,2}(x_{i,j-1}), m_{i,j-1}, r_{i,j-1}), x'_{i,j}))\right) \end{array}\right\}$$

$$\text{for } k = 1,\dots,T;\ w' = 0, 5000, 10000, \dots, B \tag{3.24}$$

3.5. Computational Results

In order to illustrate the two models, and the proposed solution procedure, we develop the representative data set shown in Table 3.1. In addition, we assume Z = \$800 as the fixed cost, R = 50% as the required reliability for Model 1 and B = \$25000 as the given budget for Model 2. We utilized both Microsoft Excel Solver and LINGO software (see www.lindo.com) to solve the DPs for each model. Excel Solver is able to solve smaller problems. It is useful to mention that nonlinear mixed-integer optimization models presented in section 3.3 have 1420 variables, 720 of which are binary and 1062 constraints, 352 of which are nonlinear.

Table 3.1. Parameters for the numerical example

Component	λ	β	α	**Failure Cost (\$)**	**Maintenance Cost (\$)**	**Replacement Cost (\$)**
1	0.00022	2.20	0.62	250	35	200
2	0.00035	2.00	0.58	240	32	210
3	0.00038	2.05	0.55	270	65	245
4	0.00034	1.90	0.50	210	42	180
5	0.00032	1.75	0.48	220	50	205
6	0.00028	2.10	0.65	280	38	235
7	0.00015	2.25	0.75	200	45	175
8	0.00012	1.80	0.68	225	30	215
9	0.00025	1.85	0.52	215	48	210
10	0.00020	2.15	0.67	255	55	250

For example, a test problem with 5 components and 20 periods took only 17 minutes on a desktop computer (Intel/Pentium 4, 3.4 GHz and 1 GB RAM). However, the example problem described above, with 10 components and 36 periods could not be solved in reasonable time. The value of objective function for the optimum solution in the Model 1 is \$18,867.13 and the system reliability under this model is 50%, (binding constraint). For the second model, the budget is equal to \$25,000 and the system reliability is maximized and found to be 60.87%. The results for these two models are presented in the Table 3.2 and 3.3 respectively.

Table 3.2. Model 1 - Maintenance and Replacement Schedule that Minimizes Total Cost (Reliability=50% and Cost=$18,867.13)

Period Component	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	M	-	-	-	R	-	-	-	-	R	-	-	M	-	M	-	-	R	-	-	-	M	-	-	R	-	M	-	-	-	-	-	-
2	-	-	-	M	-	-	-	M	M	-	M	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-
3	-	-	-	M	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	R	-	-	-	M	-	-	-	-	R	-	-	-	-	-	-
4	-	-	-	M	-	-	-	R	-	-	M	-	M	-	-	M	-	M	-	M	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-
5	-	-	-	-	-	-	-	M	-	-	M	-	M	-	-	M	-	-	-	M	-	-	-	M	M	-	-	M	-	-	-	-	-	-	-	-
6	-	-	-	M	-	-	-	-	R	-	-	-	M	-	-	M	-	R	-	-	M	-	-	M	M	-	-	-	R	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	M	-	M	-	M	-	-	M	-	M	-	-	M	-	-	M	M	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	M	M	-	-	-	M	-	-	M	-	M	-	M	-	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	-	-	-	R	-	-	-	-	-	R	-	-	-	-	-	-

Table 3.3. Model 2 - Maintenance and Replacement Schedule that Maximizes Reliability (Budget=$25,000 and Reliability=60.87%)

Period Component	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	M	-	-	-	-	-	-	R	-	-	R	-	-	-	M	-	R	M	-	-	M	-	R	-	-	M	-	R	-	M	-	R	-	-	-	-
2	R	-	-	-	-	-	-	R	-	-	R	-	-	-	M	M	R	M	-	-	M	-	R	-	-	M	-	M	M	M	-	R	-	-	-	-
3	M	-	-	-	-	-	-	R	-	-	R	-	-	-	R	-	M	M	-	-	R	-	M	-	-	M	-	R	M	M	-	R	-	-	-	-
4	M	-	-	-	-	-	-	R	-	-	M	-	-	-	M	-	R	-	-	-	R	-	M	-	-	R	-	M	M	-	-	R	-	-	-	-
5	-	-	-	-	-	-	-	R	-	-	M	-	-	-	M	-	R	-	-	-	M	-	M	-	-	R	-	-	M	-	M	M	-	-	-	-
6	M	-	-	-	-	-	-	R	-	-	R	-	-	-	M	M	M	M	-	-	R	-	M	-	-	R	-	M	M	M	-	R	-	-	-	-
7	-	-	-	-	-	-	-	R	-	-	R	-	-	-	-	-	R	-	-	-	R	-	-	-	-	R	-	-	R	-	-	R	-	-	-	-
8	-	-	-	-	-	-	-	M	-	-	M	-	-	-	-	-	R	-	-	-	M	-	M	-	-	M	-	M	M	-	-	M	-	-	-	-
9	M	-	-	-	-	-	-	R	-	-	M	-	-	-	M	-	R	-	-	-	M	-	M	-	-	M	-	M	-	-	R	-	-	-	-	-
10	-	-	-	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	R	-	-	R	-	-	-	-

Note that in both models most of maintenance and replacement actions tend to occur in the same period, which reflects the effect of the fixed cost Z. It is also interesting to note that once a maintenance or repair action occurs, it is often followed by a period of inactivity. Such observations can perhaps lead to the development of simple heuristic solution procedures in follow on work.

Another interesting aspect of this type of modeling is that one can analyze the effective age of each component. Maintenance managers could use the model to track the effective age of the components and then utilize the information to initiate additional monitoring activities. For example, after a component reaches a set effective age, additional monitoring, tests or inspections might be warranted to assist in the detection of imminent failure.

The minimum, maximum, and average effective age of each component are shown in Tables 3.4 and 3.5. Notice that the minimum effective age of each component is equal to zero at the beginning of planning horizon. Hence, minimum effective ages of components are shown from the second month on. Note that every component was replaced at some time during the planning horizon. The effective age for the components ranges from roughly 0-15 months with an average age of about 4 months. For model 2, since we considered more budget, the effective age of components is less than effective age of components in model 1 and ranges from 0-12 month with an average age about 2.9 months.

Table 3.4. Effective age of components in Model 1

Component	Minimum Effective Age (month)	Maximum Effective Age (month)	Average Effective Age (month)
1	0.0	7.2	3.0
2	0.0	6.3	3.2
3	0.0	7.2	3.1
4	0.0	6.1	3.0
5	1.0	10.5	4.9
6	0.0	7.6	3.5
7	0.0	9.0	3.8
8	1.0	15.6	7.2
9	1.0	10.3	4.8
10	0.0	9.0	3.7

Table 3.5. Effective age of components in Model 2

Component	Minimum Effective Age (month)	Maximum Effective Age (month)	Average Effective Age (month)
1	0.0	7.6	2.4
2	0.0	7.0	2.3
3	0.0	7.6	2.3
4	0.0	7.5	2.5
5	0.0	8.0	2.9
6	0.0	7.7	2.5
7	0.0	8.0	2.6
8	0.0	11.7	5.3
9	0.0	7.5	3.1
10	0.0	8.0	2.5

Figures 3.3.1 through 3.3.10 and Figures 3.4.1 through 3.4.10 show the effective age of each component. As we can see, when a component is maintained the effective age of that component drops based on the value of improvement factor, a, presented in Table 3.1. For example based on the effective age presented in Figure 3.3.1, component 1 does not receive any maintenance action at the first 3 periods, but it is maintained at the 4th period, and replaced at 8th period. This causes the effective age drops to zero and the component 1 works as a new one at the beginning of the next period. Another important feature presented in Figures 3.3 and 3.4 is the effect of failure rate on the number and frequency of maintenance and replacement actions of components over a planning horizon. For example, compare the variations in the effective age of components 7 and 8. It can be seen in Figures 3.3.7, 3.3.8, 3.4.7, and 3.4.8 that component 7 is just replaced and there is no maintenance action is performed in this component. On the other hand, component 8 is just maintained and it is not replaced, in Figure 3.4.8 it is replaced just once. This is related to values of λ and β for each component. In Table 3.1, component 7 has 0.00015 and 2.25 and component 8 has 0.00012 and 1.8 for parameters λ and β, which means that component 7 has the higher failure rate and the greater probability to fail than component 8. Therefore, it is necessary that component 7 receives more replacement actions in order to satisfy the required reliability or to maximize the of the system's reliability.

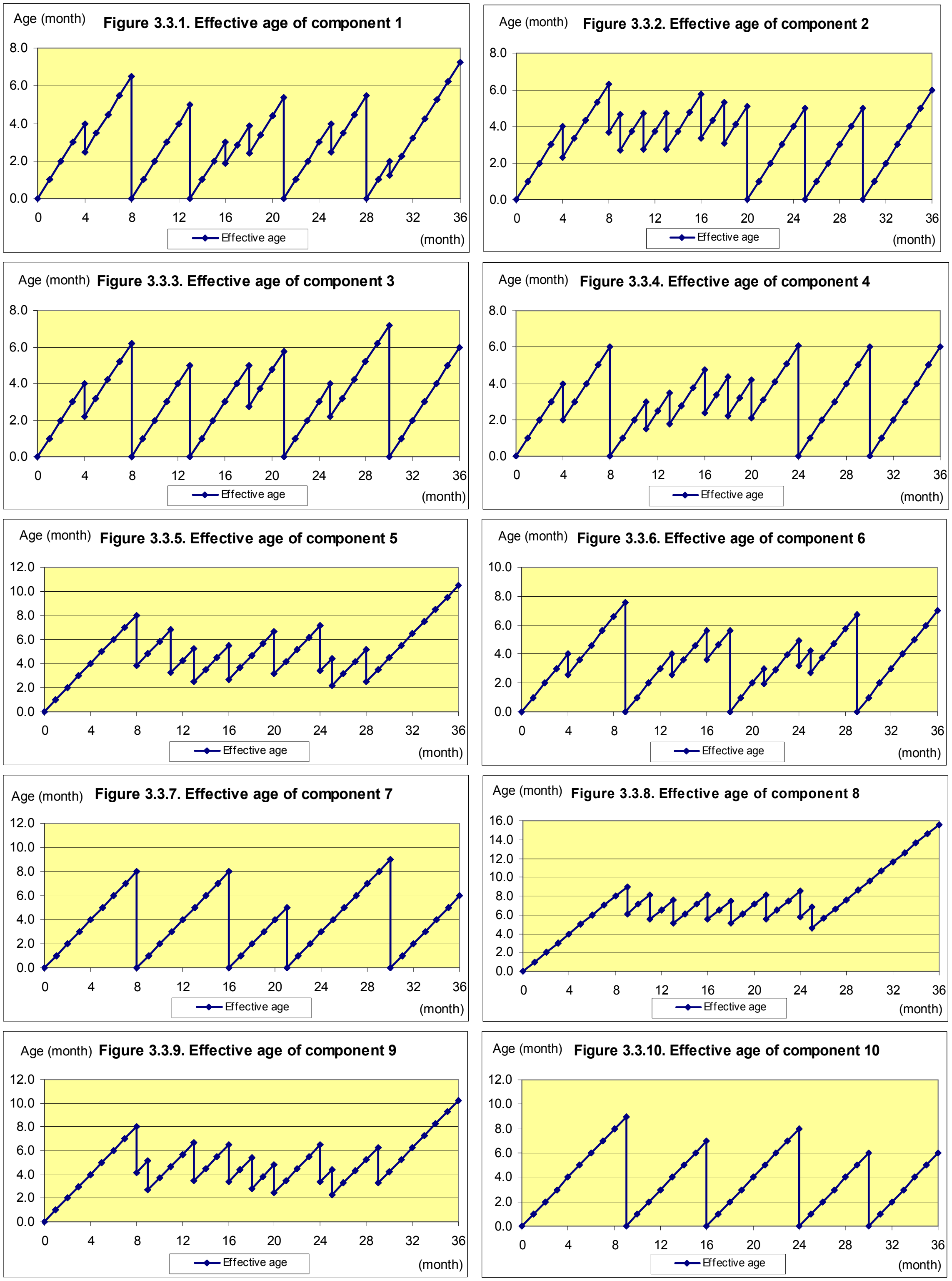
Figure 3.3.1. Effective age of component 1

Figure 3.3.2. Effective age of component 2

Figure 3.3.3. Effective age of component 3

Figure 3.3.4. Effective age of component 4

Figure 3.3.5. Effective age of component 5

Figure 3.3.6. Effective age of component 6

Figure 3.3.7. Effective age of component 7

Figure 3.3.8. Effective age of component 8

Figure 3.3.9. Effective age of component 9

Figure 3.3.10. Effective age of component 10

Figure 3.3. Effective age of components in Model 1

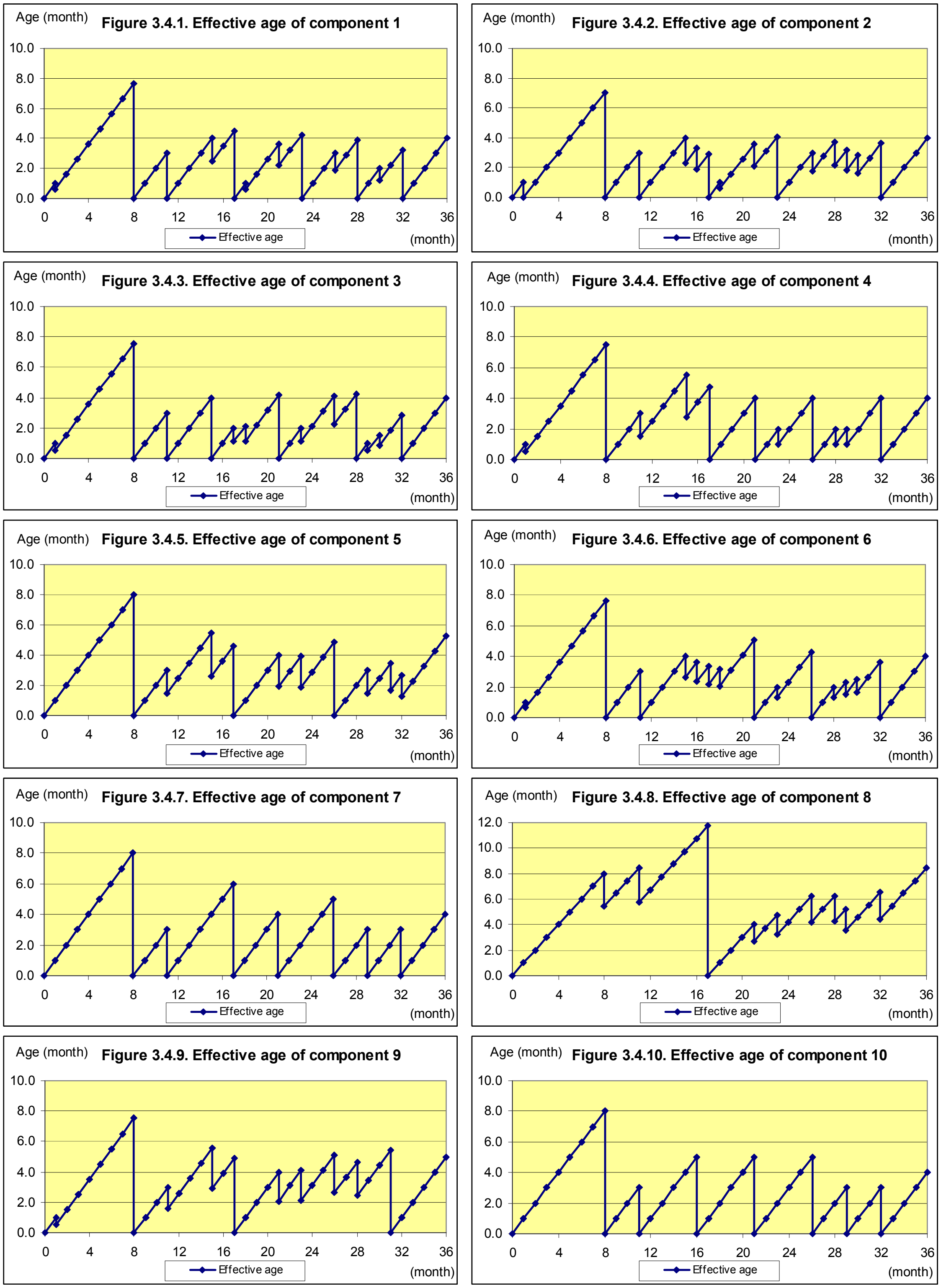
Figure 3.4.1. Effective age of component 1

Figure 3.4.2. Effective age of component 2

Figure 3.4.3. Effective age of component 3

Figure 3.4.4. Effective age of component 4

Figure 3.4.5. Effective age of component 5

Figure 3.4.6. Effective age of component 6

Figure 3.4.7. Effective age of component 7

Figure 3.4.8. Effective age of component 8

Figure 3.4.9. Effective age of component 9

Figure 3.4.10. Effective age of component 10

Figure 3.4. Effective age of components in Model 2

3.6. Chapter Summary

This chapter presented basic assumptions for the formulation of preventive maintenance and replacement strategies in order to find the best sequence of actions for each component in the system over a planning horizon such that overall costs are minimized or the reliability of the system is maximized were presented. Two nonlinear mixed integer programming models were proposed and optimized using a dynamic programming approach. The application of the optimization models was presented via numerical example and the computational results of both models were analyzed.

Chapter 4

Optimization Models - Heuristic Algorithms

4.1. Introduction

Chapter 3 presented two nonlinear mixed integer programming models that were optimized using dynamic programming. Because of the computational complexity of dynamic programming to solve real large-scale problems and its weakness to solve such problems in a reasonable time, we apply heuristic methods to tackle the problem. This chapter presents a multi-objective optimization model to find the optimal preventive maintenance and replacement schedule of multi-component systems, which is an extension of proposed models in Chapter 3. Two heuristic algorithms are adapted and modified to solve the multi-objective optimization model. Computational results show the feasibility and effectiveness of the proposed approaches.

4.2. Formulation

Based on the equations (3.9) and (3.10), we assume that the general effect of inflation increases the cost of failures over time, at a rate off $inffailure$ percent per period. Thus we find, $F_{i,j}$, the cost of failures attributable to component i in period j as:

$$F_{i,j} = F_i . \lambda_i \left(\left(X'_{i,j} \right)^{\beta_i} - \left(X_{i,j} \right)^{\beta_i} \right) (1 + inffailure)^j \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{4.1}$$

In addition, we assume a separate inflation factor, $infm$, $infr$, and $infz$ for maintenance, replacement and fixed costs increases over time, and find that the associated costs of maintenance activities of component i in period j as follows:

$$M_{i,j} = M_i (1 + infm)^j \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{4.2}$$

$$R_{i,j} = R_i (1 + infr)^j \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{4.3}$$

$$Z_j = Z(1 + infz)^j \left(1 - \prod_{i=1}^{N} \left(1 - \left(m_{i,j} + r_{i,j} \right) \right) \right) \quad \text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \tag{4.4}$$

Note that $m_{i,j}$ and $r_{i,j}$ are binary variables of maintenance and replacement actions for component i in period j and they cannot be equal to one simultaneously. The last term of the above equation mentions that if a component is maintained or replaced in each period, the whole system encounters with the defined fixed cost.

4.3. Optimization Model

From our definitions of each type of cost and by using standard time value of money concepts and an interest rate int, we can find the total net present worth (NPW) of the cost of failure, maintenance, replacement, and fixed over the T periods and we can rewrite the objective function of total cost that should be minimized. Finally, the multi-objective optimization model corresponds to the cost and reliability functions can be expressed as:

$$Min\ Total\ Cost = \sum_{j=1}^{T} \left(\left(\sum_{i=1}^{N} \begin{bmatrix} F_i.\lambda_i \left(\left(X'_{i,j} \right)^{\beta_i} - \left(X_{i,j} \right)^{\beta_i} \right) \left(1 + inffailure \right)^j \\ + M_i \left(1 + infm \right)^j .m_{i,j} + R_i \left(1 + infr \right)^j .r_{i,j} \end{bmatrix} \right) + Z \left(1 + infz \right)^j \left(1 - \prod_{i=1}^{N} \left(1 - \left(m_{i,j} + r_{i,j} \right) \right) \right) \right) \left(1 + int \right)^{-j}$$

$$Max\ Reliability = \prod_{i=1}^{N} \prod_{j=1}^{T} e^{-\left[\lambda_i \left(\left(X'_{i,j} \right)^{\beta_i} - \left(X_{i,j} \right)^{\beta_i} \right) \right]}$$

$$\begin{array}{ll}
s.t.: & \\
\quad X_{i,1} = 0 & i = 1,\ldots,N \\
\quad X_{i,j} = (1 - m_{i,j-1})(1 - r_{i,j-1}) X'_{i,j-1} + m_{i,j-1} (\alpha . X'_{i,j-1}) & i = 1,\ldots,N \text{ and } j = 2,\ldots,T \\
\quad X'_{i,j} = X_{i,j} + \dfrac{T}{J} & i = 1,\ldots,N \text{ and } j = 1,\ldots,T \\
\quad m_{i,j} + r_{i,j} \leq 1 & i = 1,\ldots,N \text{ and } j = 1,\ldots,T \\
\quad m_{i,j}, r_{i,j} = 0 \text{ or } 1 & i = 1,\ldots,N \text{ and } j = 1,\ldots,T \\
\quad X_{i,j}, X'_{i,j} \geq 0 & i = 1,\ldots,N \text{ and } j = 1,\ldots,T
\end{array} \tag{4.5}$$

4.4. Multi-Objective Genetic Algorithms

Genetic Algorithms (GA's) were developed and introduced by John Holland (1975). They have been designed as general search strategies and optimization methods working on populations of feasible solutions. Based on population search approach, GA's are able to solve multi-objective optimization problems. A generic single-objective GA can be easily modified to search a new set of multiple non-dominated solutions. The ability of GA to simultaneously search different regions of a solution space makes it possible to find a diverse set of solutions for difficult problems with non-convex, discontinuous, and multi-modal solutions spaces.

4.4.1 Representation of Solutions

In order to represent the solution of the proposed preventive maintenance and replacement scheduling problem with do nothing, maintenance and replacement actions; we define an array with length of $N \times T$ for N components and T periods where each cell in that array contains 0, 1 or 2 corresponds to three different actions.

4.4.2 Fitness Functions

Since the optimization model presented in (4.5) is a multi-objective optimization problem, we consider three different fitness functions in order to represent the problem as a single objective optimization problem.

$$Fitness_1 = w_1\left(Total\ Cost / Cost_{\max}\right) + w_2\left(-Reliability\right) \tag{4.6}$$

$$Fitness_2 = \left(-Reliability\right) + \left|Total\ Cost - Given\ budget\right| \tag{4.7}$$

$$Fitness_3 = \left(Total\ Cost\right) + Cost_{\max} \times \left|Reliability - Required\ Reliability\right| \tag{4.8}$$

Note that the above fitness functions are all subject to minimization. $Fitness_1$ is based on the weighted summation of the normalized cost function and the reliability function with the condition $w_1 + w_2 = 1$, see Cohon (1978). In order to normalize the cost function, we defined $Cost_{\max}$ as the denominator. This coefficient is the maximum amount of cost that the system could incur which is when all components are replaced in each period.

The second fitness function, $Fitness_2$, considers maximizing the reliability and minimizing a penalty term of total cost. The penalty term is based on violated values of the total cost of maintenance activities and the given budget for the system. Since the violated values have larger amount in comparison with reliability values, minimization of the penalty term has a priority to maximize of reliability.

The third fitness function, $Fitness_3$, minimizes the total cost and absolute values of subtraction of reliability and required reliability of the system. We considered

$Cost_{\max}$ as the coefficient of the penalty term in order to make a same magnitude for both parts.

4.4.3. Crossover Procedure

The crossover procedures create a new solution as the offspring of a pair of existing ones (parent solutions). The offspring inherit some useful properties of both parents in order to facilitate its propagation throughout the population.

We consider several crossover procedures as follows:

a) One-Point Crossover: In this type of crossover, first, we generate a random number between 1 and $N \times T$, then create an offspring from selected parents in which all elements located to the left of that random position are copied from the first parent and the rest of the elements are copied from the second parent.

b) Two-Point Crossover: In this type crossover, first we generate two random numbers between 1 and $N \times T$, then create an offspring from selected parents in which all elements outside the position of those random numbers are copied from the first parent and inside elements are copied from the second parents.

c) Two-Point Inverse Crossover: In this type crossover, first we generate two random numbers between 1 and $N \times T$, then create an offspring from selected parents in which all elements outside the position of those random numbers are copied from the first parent but in an inverse order and inside elements are copied from the second parents. If the chosen parents are identical, this type of crossover makes a different offspring, which is not the same to its parents.

d) N-Point Crossover: we define a new procedure for crossover in which a new offspring is made based on N components of its parents. In this type of crossover, the even components are copied from the first parent and the odd components are copied from the second parent.

e) NT-Point Crossover: In this type crossover, the even genes are copied from the first parent and odd genes are copied from the second one.

4.4.4. Mutation Procedure

The mutation procedure is applied to the offspring solution. It makes changes into the solution encoding string by modifying some of the string elements.

Based on the special structure of the preventive maintenance and replacement scheduling problem in which if even one maintenance or replacement action takes place in a period, the whole system encounters a fixed cost, we define a new type of mutation procedure. In this type of mutation, first we generate a random number between 1 and $N \times T$, then change the corresponding gene to 1 or 2 if it is equal to 0, or change it to 0 if it is equal to 1 or 2 and do same procedure in the same period for other components. This helps to create solutions in which maintenance and replacement activities tend to occur in the same periods for all components.

4.4.5. Generational GA

In the generational GA, the entire population is replaced each generation. The generational GA uses two populations at the reproduction stage. One population contains the parents to be selected and the second one is generated to hold their progeny. The generational GA algorithm is as follows, see Goldberg (1989) and Lisnianski and Levetin (2003):

```
Begin Generational GA
    g=0
    Produce initial population P(g)
    Determine the fitness values of members in P(g)
    While GA termination condition is not satisfied, do
        g=g+1
        Select solutions from P(g-1) for P(g) based on their fitness value with the
              probability of $p_{selection}$
        Make an offspring from selected parents from P(g-1) with the probability of
              $p_{crossover}$
        Mutate solutions from P(g-1) with the probability of $p_{mutation}$
        Determine the fitness values of members in P(g)
    End while
End Generational GA
```

4.4.6. Steady State GA

The steady state GA uses the same population for both parents and their progeny. When the generic operation on the parents is completed, the new offspring takes the place of the members of the previous generation within that population. The steady state GA algorithm is as follows, see Whitley (1989) and Lisnianski and Levetin (2003):

```
Begin Steady State GA
    Produce initial population P
    Determine the fitness values of members in P
    While GA termination condition is not satisfied, do
        While genetic cycle termination condition is not satisfied, do
            Make an offspring from selected parents
            Mutate the produced offspring with the probability of $p_{mutation}$
            Determine the fitness values of the new produced solution
            Replace the new produced solution with the worst solution in P if its fitness
              value is better than the fitness value of the worst solution
            Discard identical solutions in P
        End while
        Update P with new produced solutions
    End while
End Steady State GA
```

4.5. Computational Results

In order to illustrate the model numerically, and the proposed solution procedure, we used data set presented in Table 3.1 and assume Z = \$800 as the fixed cost and a 36-period planning horizon. In addition, we set the GA parameters for both generational and steady state GA as presented in Table 4.1. Finally, we consider the inflation rate of failure, maintenance, replacement, and fixed cost equal to 1%, 1.5%, 2%, and 1% respectively and 3% as the interest rate for engineering economy parameters. We utilized MATLAB R2007a (see www.mathworks.com) programming environment to develop the generational and steady state GA as well as to define the fitness functions.

Table 4.1. Parameters of Genetic Algorithms

Generational GA		**Steady State GA**	
Number of Generations	500	Number of Generations	1
Population Size	100	Genetic Cycle	500
Probability of Selection	20%	Number of Iterations	100
Probability of Crossover	40%	Population Size	100
Probability of Mutation	40%	Probability of Mutation	100%

4.5.1. Computational Results of Fitness Function 1

We run both generation and steady state GA with fitness function 1 for the set of weights for both objectives functions and achieved non-inferior solutions shown in Table 4.2. By using the new crossover and mutation procedures, we were able to solve the model in less than 2 minutes for both algorithms. Tables 4.3 and 4.4 show the optimal preventive and replacement schedule of fitness function 1 for 0.8 and 0.2 as the weights for cost and reliability objective functions. With these weights, the values of objective functions are \$6,240.55 and 62.93% via generational GA and are \$6,979.54 and 65.36% via steady state GA. It should be mentioned that all of replacement actions tend to occur in the same period, which reflects the effect of the fixed cost *Z*. It is also interesting to note that once replacement action occurs, it is always followed by a period of inactivity.

Table 4.2. Non-inferior solutions resulted from Fitness Function 1

Weights		**Generational GA**		**Steady State GA**	
W1	**W2**	**Cost**	**Reliability**	**Cost**	**Reliability**
0.0	**1.0**	$ 37,334.28	91.03%	$ 37,334.28	91.03%
0.1	**0.9**	$ 37,334.28	91.03%	$ 37,229.57	90.98%
0.2	**0.8**	$ 33,585.74	89.89%	$ 32,586.72	90.08%
0.3	**0.7**	$ 28,004.50	88.63%	$ 27,426.80	88.32%
0.4	**0.6**	$ 20,127.67	84.43%	$ 21,414.99	85.48%
0.5	**0.5**	$ 14,602.70	80.23%	$ 16,697.21	81.97%
0.6	**0.4**	$ 10,599.07	74.85%	$ 12,694.47	77.29%
0.7	**0.3**	$ 9,080.44	71.71%	$ 9,638.40	72.86%
0.8	**0.2**	$ 6,240.55	62.93%	$ 6,979.54	65.36%
0.9	**0.1**	$ 3,581.16	48.79%	$ 2,602.64	39.80%
1.0	**0.0**	$ 454.85	2.22%	$ 454.85	2.22%

Table 4.3. Maintenance and Replacement Schedule, Fitness Function 1, Generational GA (w_1=80% and w_2=20%)

Period Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
2	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
3	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
4	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
5	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
6	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
7	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
8	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
9	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
10	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-

Table 4.4. Maintenance and Replacement Schedule, Fitness Function 1, Steady State GA (w_1=80% and w_2=20%)

Period Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
2	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
3	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
4	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
5	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
6	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
7	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
8	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
9	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-
10	-	R	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	-	R	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-

Figure 4.1 represents Pareto optimal solutions (trade off curves) obtained by generational and steady state GA for fitness function 1. As it can be seen, both Pareto solutions closely correspond and the curvature of both shows that the best part of the curves is somewhere between 50% and 80% of reliability ($5000 to $15000). Figures 4.2 and 4.3 show the cost and reliability improvement in terms of number of generations and genetic cycles in generational and steady state GAs. As we can see, the convergence of the steady state GA is somewhat faster than the convergence of generational GA but the quality of final solution resulted in generational GA is slightly better than steady state GA.

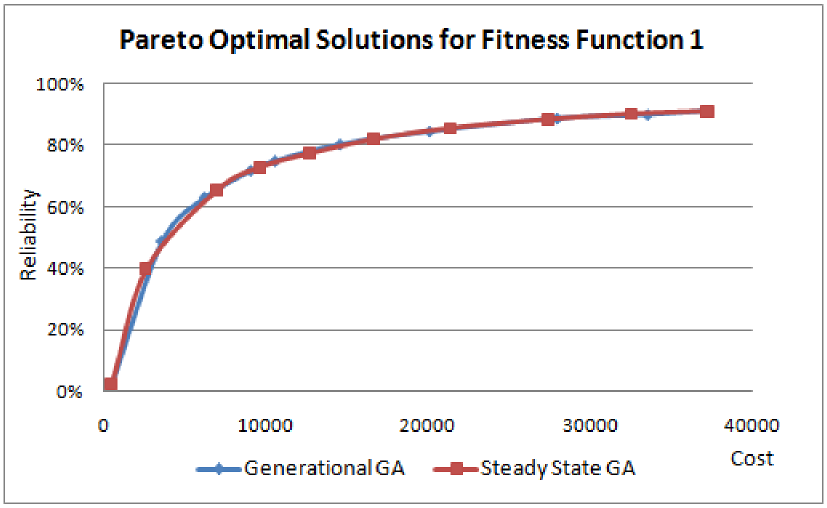

Figure 4.1. Pareto Optimal Solutions for Fitness Function 1

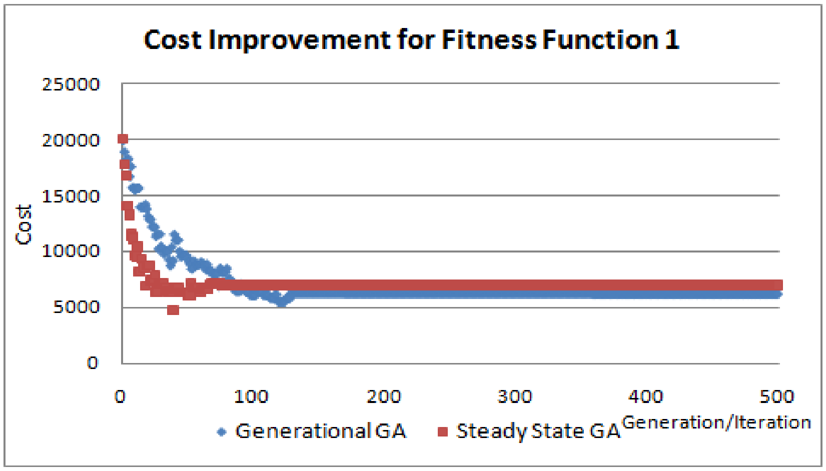

Figure 4.2. Cost Improvement for Fitness Function 1

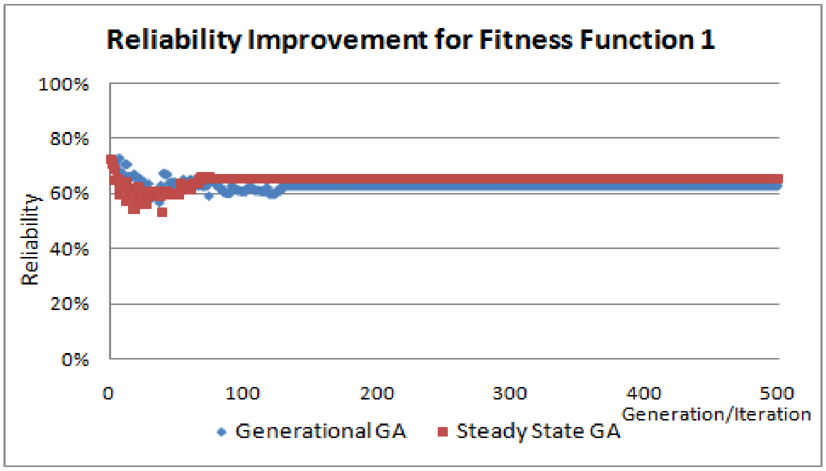

Figure 4.3. Reliability Improvement for Fitness Function 1

4.5.2. Computational Results of Fitness Function 2

We optimized the model (4.5) with fitness function 2 and by considering different budget levels for the system and obtained non-inferior solutions presented in Table 4.5. Tables 4.6 and 4.7 show the optimal preventive and replacement schedule of fitness function 2 for the given budget equal to $5000. With this budget, the reliability of the system resulted by generational and steady state GA respectively is 54.07% and 51.88% respectively. As we can see that in this situation, all of maintenance and replacement actions take place in the same period and once maintenance or replacement action occurs, it is often followed by a period of inactivity.

Table 4.5. Non-inferior solutions resulted from Fitness Function 2

Given Budget	Generational GA		Steady State GA	
	Cost	Reliability	Cost	Reliability
$ 2,000.00	$ 2,000.61	14.94%	$ 2,000.12	18.88%
$ 4,000.00	$ 4,000.23	42.14%	$ 4,000.07	35.61%
$ 6,000.00	$ 6,000.13	58.00%	$ 6,000.03	56.95%
$ 8,000.00	$ 7,999.97	64.98%	$ 7,999.87	62.38%
$ 10,000.00	$ 9,999.96	69.07%	$ 9,999.98	66.39%
$ 12,000.00	$ 11,998.88	75.24%	$ 11,999.70	72.31%
$ 14,000.00	$ 14,000.02	77.98%	$ 13,999.10	75.42%
$ 16,000.00	$ 15,999.56	80.23%	$ 16,000.65	78.92%
$ 18,000.00	$ 17,999.98	83.56%	$ 17,999.33	81.25%
$ 20,000.00	$ 20,000.40	85.12%	$ 19,999.93	83.11%

Table 4.6. Maintenance and Replacement Schedule, Fitness Function 2, Generational GA (Budget=$5000 and Reliability=54.07%)

Period Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	-	-	-	R	-	-	R	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
2	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
3	-	-	-	-	R	-	-	R	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
4	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
5	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
6	-	-	-	-	R	-	-	R	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
7	-	-	-	-	R	-	-	R	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
8	-	-	-	-	R	-	-	M	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
9	-	-	-	-	R	-	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-
10	-	-	-	-	R	-	-	R	-	-	-	R	-	-	-	-	R	-	-	-	-	M	-	-	-	R	-	-	-	-	M	-	-	-	-	-

Table 4.7. Maintenance and Replacement Schedule, Fitness Function 2, Steady State GA (Budget=$5000 and Reliability=51.88%)

Period Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	-	-	-	-	R	-	-	R	R	-	-	R	-	M	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	R	-	-	-	R	-	-	-	-	M	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	R	-	-	R	R	-	-	M	-	M	-	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-
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10	-	-	-	-	R	-	-	-	R	-	-	R	-	M	R	-	-	-	-	R	-	-	-	-	R	-	-	-	-	-	-	-	-	-	-	-

Figure 4.4 shows Pareto optimal solutions (trade off curves) obtained by generational and steady state GA for fitness function 2. As it can be seen, both Pareto solutions are relatively similar to each other. The cost and reliability improvement in terms of number of generations and genetic cycles in generational and steady state GA are shown in Figures 4.5 and 4.6. As we can see, the convergence of the steady state GA is little bit faster than the convergence of generational GA at the beginning iterations.

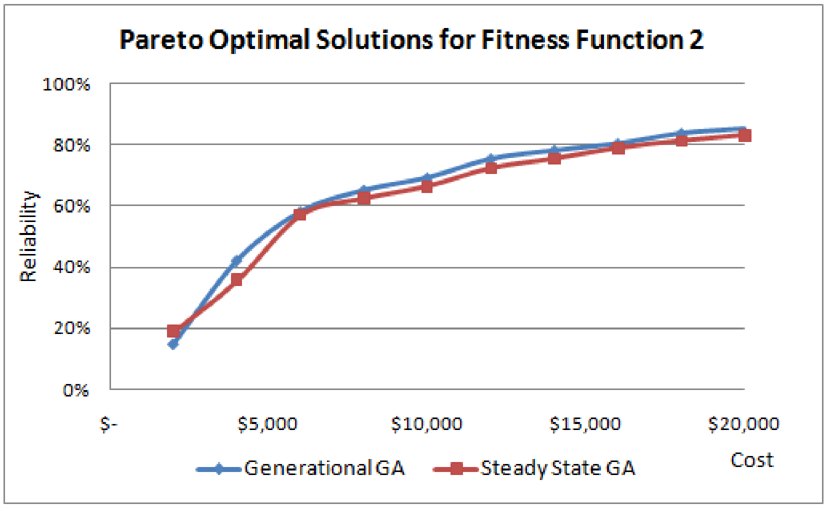

Figure 4.4. Pareto Optimal Solutions for Fitness Function 2

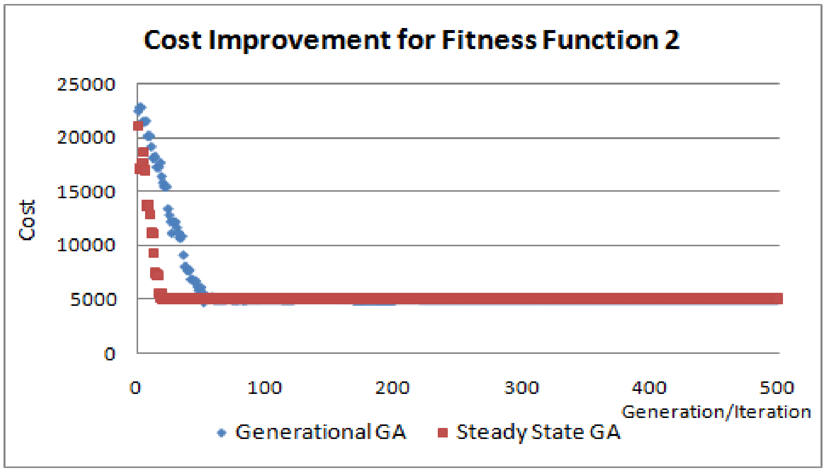

Figure 4.5. Cost Improvement for Fitness Function 2

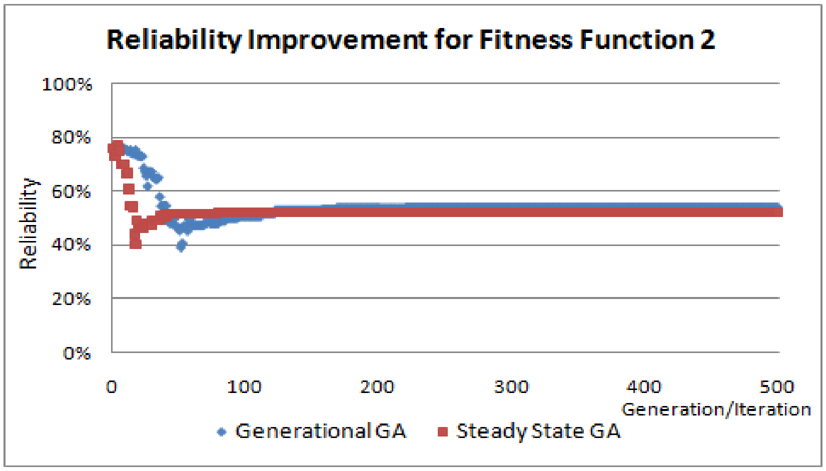

Figure 4.6. Reliability Improvement for Fitness Function 2

4.5.3. Computational Results of Fitness Function 3

Finally, Table 4.8 presents the non-inferior solutions of the model with fitness function 3 for different required reliability. Tables 4.9 and 4.10 show the optimal preventive and replacement schedule of fitness function 3 with 50% as the desired reliability. With this level of required reliability, the total cost of the system is $4109.02 and $5251.48 resulted by generational and steady state GA respectively. As it can be seen, the structure of both schedules is same as the structure found using previous fitness functions.

Table 4.8. Non-inferior solutions resulted from Fitness Function 3

Required Reliability	Generational GA		Steady State GA	
	Cost	Reliability	Cost	Reliability
0%	$ 454.85	2.22%	$ 454.85	2.22%
10%	$ 908.70	9.82%	$ 1,253.96	10.00%
20%	$ 1,544.45	20.13%	$ 1,843.41	19.88%
30%	$ 1,971.91	30.02%	$ 3,470.56	29.95%
40%	$ 3,134.55	39.94%	$ 4,407.27	39.98%
50%	$ 4,109.02	50.00%	$ 5,251.48	49.99%
60%	$ 6,381.03	59.95%	$ 7,754.48	59.94%
70%	$ 8,956.37	70.04%	$ 8,903.02	70.02%
80%	$ 14,262.18	79.81%	$ 14,455.02	79.57%
90%	$ 14,286.09	80.25%	$ 15,100.48	80.40%
100%	$ 16,076.14	81.53%	$ 15,103.18	80.67%

Table 4.9. Maintenance and Replacement Schedule, Fitness Function 3, Generational GA (Reliability=50% and Cost=$4109.02)

Period Component	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
4	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	R	-	-	-	-	R	-	-	-	-	R	-	-	-	M	-	-	M	-	-	-	R	-	-	-	-	-	-	-	-	-	-

Table 4.10. Maintenance and Replacement Schedule, Fitness Function 3, Steady State GA (Reliability=50% and Cost=$5251.48)

Period Component	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
2	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
3	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
4	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
5	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
6	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
7	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
8	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
9	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-
10	-	-	R	-	-	R	-	M	-	-	M	-	M	-	R	-	-	-	-	R	-	-	M	M	-	-	-	M	-	-	-	-	-	-	-	-

Figure 4.7 shows Pareto optimal solutions (trade off curves) obtained by generational and steady state GA for fitness function 3. In this case, the Pareto solutions do not exactly correspond to each other. Figures 4.8 and 4.9 show the cost and reliability improvement in both GAs. In this case, the convergence of both algorithms is same but generational GA reduces cost better than steady state GA. A comparison between trade off curves from the three fitness functions using the two algorithms is presented in Figure 4.10. We can conclude that fitness function 1 and fitness function 3 with generational GA give better non-inferior solutions when compared with fitness function 2 and fitness function 3 with the steady state GA.

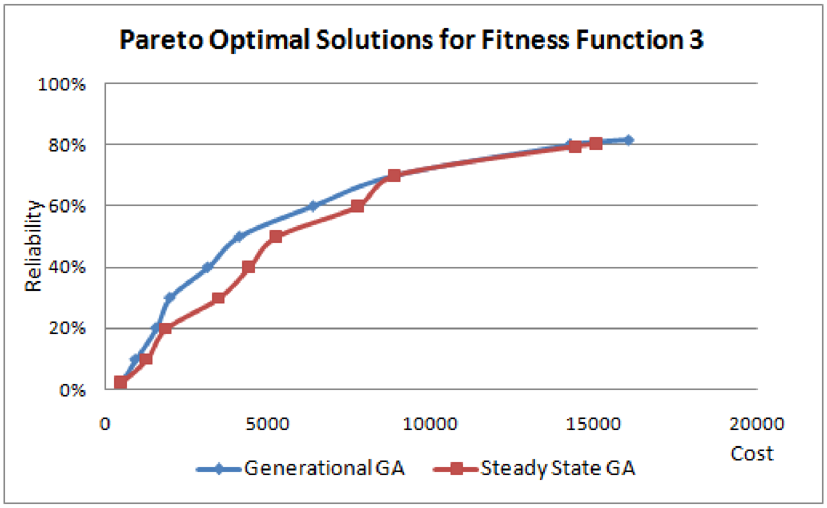

Figure 4.7. Pareto Optimal Solutions for Fitness Function 3

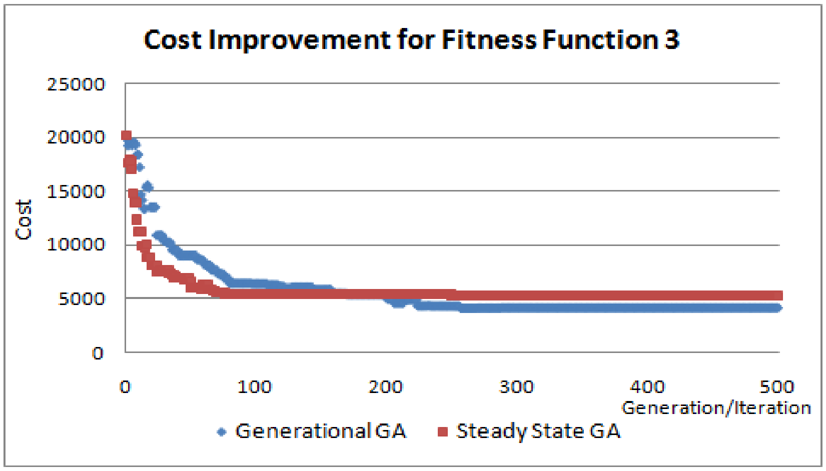

Figure 4.8. Cost Improvement for Fitness Function 3

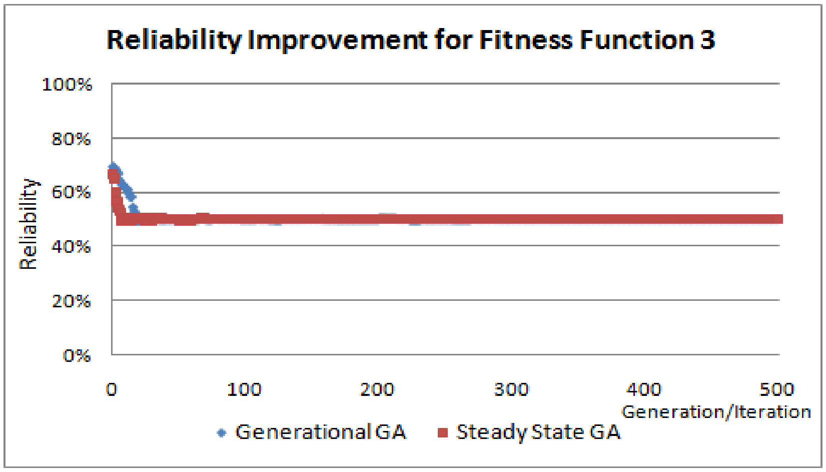

Figure 4.9. Reliability Improvement for Fitness Function 3

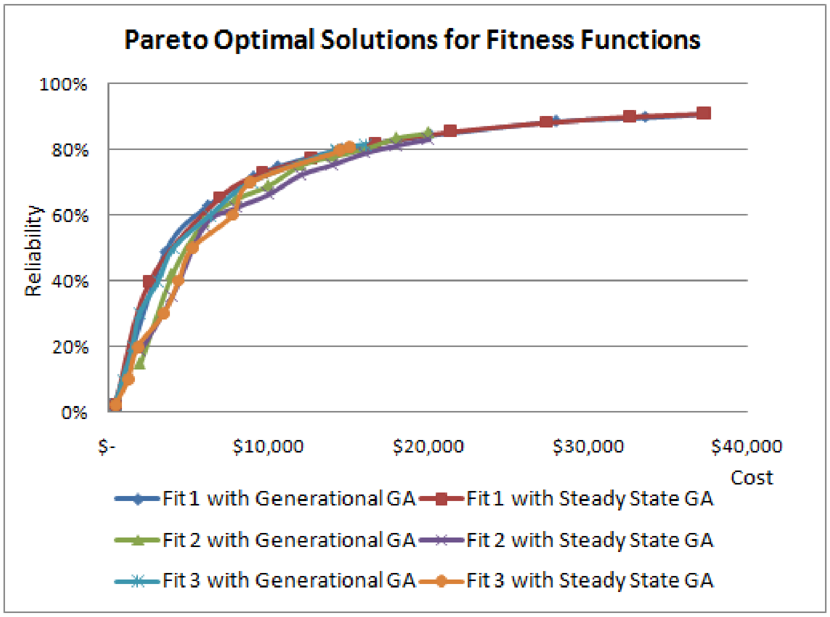

Figure 4.10. Pareto Optimal Solutions for all Fitness Functions

4.6. Chapter Summary

In this chapter, the extension of optimization models formulated in Chapter 3 was presented by considering engineering economy aspects. The new model was formulated as a multi-objective optimization model. Generational and steady state genetic algorithms were used to optimize the model and several crossover and mutation procedures were developed based on the special structure of the model. In addition, three different fitness functions were defined and utilized to achieve the best non-inferior solutions (Pareto optimal solutions). By analyzing the computational results of each algorithm with each fitness function, we could show the efficiency and effectiveness of algorithms and fitness functions. Finally, the convergence of algorithms in terms of cost and reliability improvement was demonstrated and examined.

Chapter 5

Research Plan

As presented in Chapter 1, the remainder of the Ph.D. dissertation will progress as follows. Implementation of multi-objective simulated annealing in order to solve the multi-objective optimization model and compare the computational results with genetic algorithms and dynamic programming results is the next step. This step completes Chapter 4, optimization models-heuristic algorithms, of this dissertation proposal and it will be finished on December 2008.

Analytical and statistical models will be constructed in order to estimate the age reduction and improvement factor parameters of the optimization models. This step will be Chapter 5 of the dissertation and it will be finished at the end of the summer 2009. The final stage includes examining the application of the developed models in a real case study. A comparison between current policies and optimal preventive maintenance and replacement schedules will be presented and feasibility and effectiveness of the models will be investigated. If the related data of that case study is available in a proper time, the dissertation can be completed at the end of 2009.

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