Strategies for Encouraging Students to Persist When Working on Cognitively-Demanding Tasks

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Two related projects

- **Task Types and Mathematics Learning**
  We investigated the ways that particular types of mathematics classroom tasks create different opportunities for students and different challenges for teachers.

- **Encouraging Persistence Maintaining Challenge**
  We are exploring what is needed to encourage students to embrace challenges and to persist even when tasks are difficult.
A helpful framework in both projects was proposed by Stein, Grover, and Henningsen (1996).
Our goals were to explore:
- how the tasks respectively contribute to mathematics learning
- features of successful exemplars of each type
- constraints which might be experienced by teachers
- teacher actions which can best support students’ learning
1. Purposeful Representational tasks:

- Teachers use tasks which involve the introduction to, or use of models, representations, tools, or explanations, which exemplify the mathematics.
  - Such tasks are associated with good traditional mathematics teaching (see Watson & Mason, 1998). The mathematical purpose is clear and the tools/models/representations are linked directly and explicitly.
2. **Contextual Tasks:**

- Teachers situate mathematics within a contextualised practical problem where the motive is explicitly mathematics.
  - This task type has a particular mathematical focus as the starting point and the context exemplifies this. The context serves the twin purposes of showing how mathematics is used to make sense of the world and motivating students to solve the task.
3. **Open Ended Tasks:**

- Students investigate specific mathematical content through open-ended tasks
  - Content specific open-ended tasks have multiple possible answers, they prompt insights into specific mathematics through students seeing and discussing the range of possible answers
A small sub-project

- To facilitate reflection and discussion on the differences between the tasks types through project teacher implementation of three tasks — one of each type involving substantively the same mathematical content.
- To collect data from the teachers that highlight the constraints and pedagogies of the different tasks types.
- To obtain data on student preferences that is linked to the teacher
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Inside and Outside the Square

For each of the following shapes find the area and perimeter.

What do you notice? Why might that be?
For each of these shapes find the perimeter and area. What do you notice? What is the relationship between perimeter and area of these types of shapes?
Dido’s Problem

Task Description
Tell students about Dido, founder and first Queen of Carthage. History tells that she arrived on the coast of North Africa, having fled from her brother King Pygmalion. She asked the local inhabitants for a small amount of land for a temporary refuge. They agreed when she asked for only as much land as could be surrounded by an ox hide. She cut the ox hide into fine strips which enabled her to encircle a very large area, which later became the city of Carthage.

Today, we are going to investigate the largest area which can be enclosed by a fixed perimeter.
The task of interest today: [We don’t have enough ox hides]

What is the longest continuous strip you can make from a Mintie wrapper without tearing it into more than one piece, and what is the greatest area which can be enclosed by a length of string that long?
Invite students to tear the Mintie wrapper carefully, trying to make it one continuous long strip. Now invite them to measure the length of their streamer. The students then cut off a piece of string that long.

Now they should take that length of string and form closed shapes with that perimeter on dot paper. For each shape, they should draw it on dot paper, recording the perimeter (the same each time hopefully) and the area, if they can determine it, either by counting squares or some other means.
Imagine that the letter L is drawn on a large piece of squared paper. Its area is 100 cm$^2$. What might be its dimensions?

A rug has an area of 2 m$^2$. What might be the dimensions? What are the perimeters of the rugs you have drawn? (try to design a number of different rugs)

(The goal is for students to realize that different shapes can have the same area but different perimeters, and to develop skills in calculating area and perimeter.)
Students make the choice of what to draw
Data Collected

- Teacher questionnaire before and after the lesson sequence
  - Before - Focus on intentions, ordering of tasks, expected challenges
  - After – Focus on adaptations, preferences, challenges
- Some observations
- Student feedback
  - Liked to do
  - Learned the most from
  - Found the easiest
- Student work samples
Students' feelings about three lessons

- Liked most
- Learned most
- Easiest

Legend:
- Dido's Problem
- What the L?
- Inside & Outside the Square
<table>
<thead>
<tr>
<th>Teacher data</th>
<th>Student data</th>
<th>Student data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pref for teaching</td>
<td>Student learning</td>
<td>Liked the most</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dido</td>
</tr>
<tr>
<td>H</td>
<td>Dido</td>
<td>14</td>
</tr>
<tr>
<td>A</td>
<td>What the L</td>
<td>20</td>
</tr>
<tr>
<td>F</td>
<td>Dido</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>What the L</td>
<td>11</td>
</tr>
<tr>
<td>C</td>
<td>Dido</td>
<td>13</td>
</tr>
<tr>
<td>E</td>
<td>Dido/Inside</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>Dido</td>
<td>4</td>
</tr>
<tr>
<td>G</td>
<td>Dido</td>
<td>8</td>
</tr>
</tbody>
</table>
Seven People went Fishing:

If the mean number of fish caught was 5, the median was 4, and the mode was 3, how many fish did each person catch?
<table>
<thead>
<tr>
<th>Task</th>
<th>Task Type</th>
<th>Task favourite</th>
<th>2nd favourite</th>
<th>Task best for learning</th>
<th>2nd best for learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A sentence with 5 words</td>
<td>3</td>
<td>4.0%</td>
<td>12.0%</td>
<td>8.0%</td>
<td>4.2%</td>
</tr>
<tr>
<td>Rock paper scissors</td>
<td>3</td>
<td>22.0%</td>
<td>27.8%</td>
<td>0.0%</td>
<td>4.2%</td>
</tr>
<tr>
<td>Seven people went fishing</td>
<td>3</td>
<td>10.0%</td>
<td>4.0%</td>
<td>18.0%</td>
<td>12.5%</td>
</tr>
<tr>
<td>Conducting a survey</td>
<td>3</td>
<td>2.0%</td>
<td>2.0%</td>
<td>2.0%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>48</td>
</tr>
</tbody>
</table>
Seven People went Fishing:

If the mean number of fish was 5, the median was 4, and the mode was 3, how many fish did each person catch?

- This is an open-ended question and a number of the student responses suggested that they valued the openness and choice:
  - You were able to make different answers and you had to find out the answer.
  - I liked the way how you had to think about every decision you make it was very fun.
  - At the start I didn’t get it but then I finally got the hang of it and got of a lot of answers and learned something new.
Summary

The data reported here support the view that there is a diversity of beliefs across students in terms of preferences for different types of tasks, in terms of their contribution to motivation, enjoyment and learning. It highlights the importance of providing a wide range of types of tasks in a variety of different sequences, in order to meet the needs of as many students as possible.
The second project

Encouraging Persistence Maintaining Challenge (EPMC)
We are exploring what is needed to encourage students to embrace challenges and to persist even when tasks are difficult.
The notion of persistence is encapsulated in widely used principles of effective teaching that recommends teachers communicate high expectations to students, which involves posing challenging tasks, and adopting associated pedagogies such as encouraging students to task risks in their learning, to justify their thinking, to make decisions, and to work with other students (Stein, Smith, Henningsen & Silver, 2009; Sullivan, 2011).
The project was founded on a belief that while it is possible for everyone to learn mathematics, it takes concentration and effort over an extended period of time to build the connections between topics, to understand the coherence of mathematical ideas, and to be able to transfer learning to practical contexts and new topics.
● Struggle is important for students if real learning is to take place. As Hiebert and Grouws (2007) noted, “we use the word *struggle* to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do *not* use *struggle* to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems” (p. 387).
Pogrow (1988) warned that by protecting the self-image of under-achieving students through giving them only “simple, dull material” (p. 84), teachers actually prevent them from developing self-confidence. He maintained that it is only through success on complex tasks that are valued by the students and teachers that such students can achieve confidence in their abilities. There will be an inevitable period of struggling while the students begin to grapple with problems but Pogrow asserted that this “controlled floundering” is essential for students to begin to think at higher levels.
A design research approach

- In the overall project, as well as in this aspect, we are adopting a design research approach which “attempts to support arguments constructed around the results of active innovation and intervention in classrooms” (Kelly, 2003, p. 3). The key elements for us are that we are intervening to prompt (possibly) different pedagogies from those used normally, our approach is iterative in that subsequent interventions are based on the findings from previous ones, and the intent is that findings address issues of practice.
As well as practical results and theoretical results, the design had the potential to contribute to the professional development of its participants (McKenney & Reeves, 2012)
Music Cards

16 songs | MySongs Music Card | $24

12 songs | MyTunes Music Card | $20

Which card is the better value?
Give two different mathematical justifications for your answer.
Four for four improves the average price

My opinion:
12 songs for $20 is more than $1 per song and for the 16 songs for $24 is 4 more songs for $4, and when in card 1 the songs are more than $1, then card 2 is cheaper, for more songs.
Unitising (per song)

$20 \div 12 \text{ songs } = \$1.66$

$24 \div 16 \text{ songs } = \$1.50$

Divide the money by the songs and the cheapest one will be your answer.
Unitising (per 48 songs)

well \[ 16 \times 3 = 48 \]

so \[ $24.00 \times 3 = $72.00 \]

then 12 goes into 48 4 times

so \[ $20.00 \times 4 = $80.00 \]
Compares to Pod Tunes

\[
\begin{align*}
\frac{16}{24} + \frac{8}{24} &= \frac{24}{24} = 1 \\
&= 1 \\
&= 1 \times 16 = 16 \\
\$1.50 &\times 16 = 24 \$1.50 \\
\$9.00 &\times 12 = 144 \$3.00 \\
\$15.00 &\text{ is less than } \$2.00 \\
\$24.00 &\text{ better value than } \$18.00
\end{align*}
\]

The 1st one is better value because the songs on it are $1.50 each but on the 2nd one the songs are more than $1.50 but less than $2.00 so it must better to get the 1st even though you spend more money on it it's better value.
Incorrect use of remainders

$1.50 each

\[ \frac{1.8}{12} = \frac{1}{2} \]

\[ \frac{1.8}{12} = \frac{1}{2} \]

\[ \frac{1.50}{12} = \frac{1}{2} \]

\[ \frac{1.50}{12} = \frac{1}{2} \]
Incorrect use of remainders

16 | 24   12 | 20
1.8  1.8

$1.80 each
Additive Thinking

The Same!

because the difference between the MySongs music card & MyTunes music card is 4. And the difference between the money is 4. The difference between the songs & the money is 8 on both. (12&20 and 16&24)
At different points in the project, we have collected information from teachers on their experiences. Prior to and after teaching the tasks, we sought teacher perceptions on strategies to encourage persistence on challenging tasks.
Sometimes when students struggle with a mathematics task, they choose not to persist. What kinds of things do you believe a teacher could do in the planning stage of a lesson and during the lesson that would help those students to persist? Please record as many as you can.

- In the planning stage, teachers could ...
- During the lesson, teachers could ...
### Most common new strategies in the planning stage for encouraging persistence

<table>
<thead>
<tr>
<th>Strategy in the planning stage</th>
<th>Number of teachers out of 35</th>
<th>Illustrative comments</th>
</tr>
</thead>
</table>
| Differentiation              | 10                            | • Have the prompting questions already to use during the session, rather than waiting for a particular misunderstanding to occur  
• Actually including enabling and extending prompts in my planning |
| Nature of tasks              | 7                             | • More problem solving activities. Plan more tasks that they need to think about instead of telling them what was wanted  
• Providing tasks which focus on a particular concept, but in the problem solving format |
| Holding back                 | 3                             | • Not telling them what to do  
• Not planning to 'teach' the concept first but waiting for the need to arise. Purposeful learning |
## Most common new strategies during the lesson for encouraging persistence

<table>
<thead>
<tr>
<th>Strategy during the lesson</th>
<th>Number of teachers out of 35</th>
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</table>
| Discussion/questioning/sharing | 11                          | - Asking lots more questions e.g. so where could you go from there? Can you explain how you got here? What could you do next? Are you sure that's correct?  
- Students share more of their thinking more of the time. Students are learning more from sharing with each other, rather than listening to me. |
| Holding back                | 10                          | - I model less at the beginning of lessons.  
- I probably don't 'over teach' during working time and share time - I allow for students to discover the mathematical concepts and strategies |
| Culture                     | 9                           | - I am a lot more willing to say to a student "I know this is hard, I want it to be hard you need to go and think a bit more about (some specific context).  
- Using phrases such as yes this is hard, zone of confusion, I want you to have a go first, I'm not going to help you for 10 minutes, prove it to me, how do you know it is correct? |
Primary – In the planning stage

(“students” removed)
Secondary – In the planning stage ("Students" removed)
Primary and Secondary

In the planning stage
Primary – During the lesson
(“students” removed)
“Time” was linked to strategies such as giving students more time to work on tasks, more thinking time, time to share with other students and teachers spending less time telling students what to do.
Secondary – During the lesson
("students" removed)
Primary and secondary-
During the lesson
Comments made about “Time”

- Sit in the zone for a longer period of time
- Time to think
- More time for enquiry learning
- Less teacher talk time
- Allowing time for students to solve the problems without interfering
- Don’t over teach during working time
- Giving them time to discuss with other children
- Students share more of their thinking more of the time
- Making and trying to allocate time to the summary phase
- Give more time to the share/summary [phase]
- Allow students thinking time
- Thinking time (x 2)
- Question time
- Discussion time
In summary, we do differently:

- holding back from telling;
- allowing some time for students to think by themselves and work on the task;
- allowing students to share ideas and work collaboratively;
- discussing the behaviours of persistence (being in the zone of confusion, risk taking, not giving up, facing the challenge);
In summary, we do differently:

- providing challenging problem solving tasks that engage students;
- planning for and using enabling and extending prompts; and
- working through the tasks in advance of the lesson.
A man goes into a shop and says to the owner: “give me as much money as I have with me and I will spend $10”. It is done, and the man does the same thing in a second and third store, after which he has no money left. How much did he start with?
We read the problem and thought hard. Most people thought $30, so we made lots of $10 notes and acted out the problem with one man and three shop owners.

\[
30 + 30 = 60 \quad 10 = 50
\]

\[
50 + 50 = 100 - 10 = 90
\]

\[
90 + 90 = 180 - 10 = 170
\]
$20 + 20 = 40 - 10 = 30$

$30 + 30 = 60 - 10 = 50$

$50 + 50 = 100 - 10 = 90$

$15 \cdot 15 + 15 = 30 - 10 = 20$

$20 + 20 = 40 - 10 = 30$

$30 + 30 = 60 - 10 = 50$
We saw that as we went along with each problem the numbers were getting bigger.

All the numbers are too big — well try 10

$\$10 \quad 10 + 10 = 20 - 10 = 10$

$10 + 10 = 20 - 10 = 10 \quad \text{We are standing still}$

$10 + 10 = 20 - 10 = 10$
Ten is the closest so far but we need less left over.

What if we try $0$?

$0 + 0 = 0 - 10$. We can’t do this because there isn’t enough. You can’t spend $10$ if you don’t have anything.
5 Could be the answer because
5 + 5 = 10 - 10 = 0
This would have to be at the last shop.

We'll have to work backwards.
Shop
3.

5 + 5 = 10 - 10 = 0
Our answer will need to be 5

\[ \square - 10 = 5 \]

\[ \square - 10 = 5 \]

We will have to use 50¢ to halve $15

$7.50 + $7.50 = 15$

15 - 10 = 5.
We need $7.50 for our answer.
$7.50 + $10 = $17.50.
$17.50 - $10 = $7.50.

$17.50 is close to $18.
$\frac{1}{2}$ of $18 = 9.$
We have 50 cents too much.
We'll have to take something from each $9.00 to equal 50 cents.
We'll need to take 25 cents from each.
$9.00 - 25 cents = 8.75$

We found out that the man started with $8.75.
It took a lot of thinking and working out.
Danny and Tom

- Tom and Danny travelled from Acton to Beamsville on foot. Danny walked half the distance and ran half the distance. Tom walked half the time and ran half the time. They started at the same time, and walked at the same speed as each other and ran at the same speed as each other. Who arrived first, or was it a tie?
Some general strategies for encouraging persistence as students work on challenging mathematics problems

It seems desirable that:

- the nature of the task is explained to the students (e.g., if there are multiple possible solutions, this can be mentioned);
- the ways of working are explained to the students, including the type of thinking in which they are expected to engage and what they might later report to the class;
- the classroom climate encourages risk taking, teachers expect the students to succeed, errors are part of learning, and students can learn even if they do not complete the task;
Some general strategies for encouraging persistence as students work on challenging mathematics problems

It seems desirable that (ctd.):

- the teacher communicates enthusiasm about the task, including encouraging the students to persist with it;
- students have a choice of ways that they can approach the task, and perhaps on the level of difficulty of the task itself;
- the class is organised to ensure that students have adequate time to work on the challenging task, including preparing some aspects such as group membership beforehand;
Some general strategies for encouraging persistence as students work on challenging mathematics problems

It seems desirable that (ctd):

- processes and expectations for recording are made clear;
- teachers are alert to incidental opportunities to explore mathematics that arises;
- some attempt is made to connect the task with the students’ experience;
Some general strategies for encouraging persistence as students work on challenging mathematics problems

It seems desirable that (ctd):

- the teacher moves around the class making regular contact with students, asking questions of the individuals or groups; and

- there is time allowed for lesson review so that students see the strategies of other students and any summaries from the teacher as learning opportunities.
The term *persistence* is used to describe the category of student actions that include concentrating, applying themselves, believing that they can succeed, and making an effort to learn.

Persistence involves:
- Spending time on tasks, even if the solution pathway is not clear
- Continuing to work on a task even if the solution pathway has been identified or the problem is already solved
- Being willing to listen to others and to explain your own ideas
- Working on tasks that are not necessarily immediately of interest
Challenging tasks require students to

- plan their approach, especially sequencing more than one step;
- process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways;
- choose their own strategies, goals, and level of accessing the task;
- spend time on the task and record their thinking;
- explain their strategies and justify their thinking to the teacher and other students.