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NOTE

Estimate of Jupiter's Deep Zonal-Wind Profile from Shoemaker–Levy 9 Data and Arnol'd's Second Stability Criterion

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A number of one- and two-layer atmospheric models have been applied to the study of Jupiter's tropospheric circulations over the past two decades in an effort to characterize the basic-state properties of the zonal winds and to study the influence these winds have on the dynamics of long-lived vortices like the Great Red Spot. By singling out the basic state that corresponds to neutral stability with respect to Arnol'd's second stability criterion, as motivated by the results of a vortex-tube stretching analysis of the Voyager wind data and by fixing the stratification parameter using the $c = 454 \pm 20$ msec^{-1} gravity wave speed determined from the Shoemaker–Levy 9 impacts, we arrive at a unique model with no free parameters. The speed and direction of the zonal wind as a function of latitude for the model's lower layer is thus determined, which roughly indicates the circulation in Jupiter's interior below the water clouds. Predictions are that the westward jets change little with depth but that the eastward jets become stronger by 50–100%. © 1995 Academic Press, Inc.

1. Introduction. Two seemingly contradictory facts stand out in the study of Jupiter's tropospheric circulations. On the one hand, the planet's zonally averaged NH_3 cloud-top motions show a complicated series of alternating eastward and westwards jets that display no change over time scales longer than a decade, based on the fact that the zonal winds obtained in 1979 by tracking small cloud features in Voyager images are identical to those obtained in 1992 using Hubble Space Telescope (HST) images. This rock-steady stability of the cloud top winds is made more remarkable by the fact that Jupiter's winds are quite zonal, especially when compared to the large eddy fluctuations found in Earth's atmosphere. On the other hand, Jupiter's winds strongly violate every shear stability theorem found in current dynamic meteorology textbooks.

One way out of this paradox has been suggested by a vortex-tube stretching analysis of the Voyager wind data for Jupiter's Great Red Spot and White Oval BC, which indicates that Jupiter's winds are in a neutrally stable configuration with respect to Arnol'd's second stability criterion. This is an obscure theorem that is not yet found in textbooks, but is nonetheless emerging as an important criterion for planetary shear flows. See Dowling (1995) for a review.

The purpose of this note is to complete the $1\frac{1}{2}$-layer quasigeostrophic model of Jupiter (described below) by fixing the model's single free parameter using the gravity wave speed obtained from the HST analysis of Shoemaker–Levy 9 (SL9) impact sites. Given that the model then has no free parameters, one may ask what predictions it makes. The answer is that it yields the zonal winds as a function of latitude for its lower layer. From a theoretical point of view this gives an estimate of the barotropic (height independent) component of Jupiter's circulations, which is a more relevant target for interior modelers than trying to reproduce the NH_3 cloud-top winds, and from an engineering point of view it indicates the sign and magnitude of the vertical wind shear below the cloud tops, which is relevant to the planning of multi-latitude entry probes.

2. Justification of the $1\frac{1}{2}$-Layer Model. The HST observations of the SL9 comet impact sites on Jupiter (Hammel *et al.* 1995) detected a transient, narrow dark ring around each of the five impact sites A, E, G, Q1, and R that expanded outward at the rate $c = 454 \pm 20$ msec^{-1} and a faint inner ring around sites E and G that expanded at a rate somewhere between 180 and 350 msec^{-1}, depending on poorly constrained assumptions about when and how the inner ring was started. Prior to the impacts in July 1994, two papers attempted to predict the nature of gravity waves emanating from the impact sites, both with partial success. The first paper, Harrington *et al.* (1994), concentrated on the global response of Jupiter's atmosphere to the impacts. They correctly predicted that there would be no long-lived vortices resulting from the impacts, but that gravity waves would be observable and yield new information about the stratification of the atmosphere. In this model the $T(p)$ profile model was extended below Jupiter's cloud tops by assuming a dry adiabatic temperature lapse rate. As discussed below, a more appropriate model would have been to use a moist adiabatic lapse rate in the vicinity of the H_2O clouds to simulate the added stability caused by the release of latent heat from rising from condensing moist air. Without the tropospheric buoyancy waveguide caused by the moist adiabat, the Harrington *et al.* gravity waves traveled in the form of a train of ripples similar to the waves one generates by throwing a rock into a pond, but not similar to the widely spaced, discrete rings observed by HST. A second shortcoming of the Harrington *et al.* model was insufficient vertical resolution. The global model was restricted to only 5 vertical layers, and this had the adverse effect of artificially lowering the top of the model to 16 mbar, which we now know caused an erroneous factor of two reduction in the speed of the model's stratospheric gravity waves over the true speed. As a result, the reported 400 msec^{-1} model wave speed matches the 454 msec^{-1} observed speed only by coincidence. We have since verified this with a 50-layer local version of the model where the top is moved to 0.001 mbar, in which case the stratospheric gravity waves travel at twice the speed of the $c = 454$ msec^{-1} wave observed by HST.

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The second prediction paper, Ingersoll *et al.* (1994), concentrated on a local model with continuous but parametric vertical structure and emphasized the effects of Jupiter's H_2O cloud on the gravity waves. In particular, they found that gravity waves are completely controlled by the tropospheric waveguide formed by the H_2O cloud, regardless of whether the comet explosions occur inside the tropospheric waveguide, near it, or far above it in the stratosphere. By assuming a moist adiabatic temperature lapse rate based on the solar O/H ratio, Ingersoll *et al.* predicted that the gravity waves would travel as discrete rings and that the leading wave would have a speed of 130 msec^{-1}. After the impacts, Ingersoll and Kanamori (1995) adjusted the model's H_2O abundance by increasing it by an order of magnitude to yield the observed 454 msec^{-1} speed, using the fact that the waveguide-controlled gravity wave speed varies as the square root of the H_2O abundance.

The discrete rings observed by HST from the SL9 impacts are good news for the $1\frac{1}{2}$-layer model we consider in this paper, because they suggest that a tropospheric waveguide is acting to create discrete normal modes. Achterberg and Ingersoll (1989) anticipated this fact and developed a detailed normal-mode representation of Jupiter's atmosphere in which they found that many large-scale phenomena could be modeled by summing over just the first few modes. In addition, they found that the modes associated with the H_2O cloud are only slightly modified if one includes the weaker effects of the NH_3 and NH_4SH clouds. The simplest modal expansion that allows for vortex-tube stretching includes just the barotropic and first baroclinic mode and is equivalent to a two-layer model in which the upper layer is active and the lower layer contains horizontally varying winds that do not change with time. Such a model is called a "$1\frac{1}{2}$-layer" model because the lower layer is specified a priori and is not free to evolve. This is justified physically on the grounds that the lower layer is much more massive than the upper layer and is therefore not affected by the weather in the upper layer. Other names for this model are "equivalent barotropic" and "reduced gravity."

It is important to note that the details of how the tropospheric waveguide is actually formed, including whether or not Jupiter's atmosphere has a water abundance that is 10 times solar, do not directly affect the results reported here. The question of what controls the static stability structure in Jupiter's atmosphere is an area of active research, and the SL9 data and the upcoming Galileo atmospheric probe will undoubtedly stimulate further investigations into this important topic.

The winds are constant with height in each model layer and we need to initialize the upper layer with a zonal-wind profile that appropriately characterizes the entire region between Jupiter's tropopause and H_2O cloud. Since we have accurate wind measurements only for the NH_3 cloud-top level, we need to either employ these winds directly with some justification or come up with a plausible modification to use as our initial condition for the model's upper layer. While we know from application of the thermal-wind equation to the Voyager IRIS infrared data that the winds decay with height in the upper troposphere and tropopause region (Flasar 1986), this information is not directly applicable here because of the crudeness of our model—the only extent to which stratospheric structure is captured is that the planet's tropopause corresponds to the free surface of the model's upper layer. However, with regard to the lower troposphere there is some evidence that the vertical wind shears are not too large. For example, Flasar and Gierasch (1986) concluded from a study of mesoscale wavetrains in the Voyager images that the Richardson number, which measures the ratio of stratification to vertical wind shear, $Ri = N^2/(d\overline{u}/dz)^2$, is of order unity in a wave duct hypothesized to exist below the NH_3 cloud tops. The NH_3 cloud tops occur around the 670-mbar level and fortuitously sample the wind speeds in the relatively low-stratification region that lies sandwiched between the high-stratification regions of the stratosphere and the H_2O cloud. It is therefore reasonable to suppose that the NH_3 cloud-top winds occur in a region of small vertical shear and are representative of the tropospheric winds. In what follows we initialize the upper layer directly with the NH_3 cloud-top zonal wind profile as determined by Limaye (1986).

TABLE I
Basic-State Assumptions for $1\frac{1}{2}$-Layer Models

Reference	$\overline{u}_2$	$\overline{q}_y L_d^2$
Ingersoll and Cuong (1981)	$\overline{u}$	$(\beta - \overline{u}_{yy})L_d^2$
Williams and Yamagata (1984)	0	$(\beta - \overline{u}_{yy})L_d^2 + \overline{u}$
Marcus (1988)	$(\beta - \overline{u}_{yy})L_d^2 + \overline{u}$	0
Dowling (1993)	$(\beta - \overline{u}_{yy})L_d^2$	$\overline{u}$

3. Jupiter's basic state. We must now complete the specification of the $1\frac{1}{2}$-layer model by determining an appropriate zonal-wind profile for the lower-layer winds. In the following discussion λ is latitude in radians and $y = (\lambda - \lambda_0)R$ is latitude in units of length measured relative to a local reference latitude λ_0, where R is the radius of the planet (the local meridional radius of curvature for Jupiter's oblate surface). The governing equation for large-scale atmospheric dynamics is conservation of potential vorticity, $Dq/Dt = 0$, and gradients in the basic-state potential vorticity are central to the theory of waves and stability. The basic-state potential vorticity gradient, $\overline{q}_y$, for the upper layer in the $1\frac{1}{2}$-layer quasigeostrophic model may be written

$$\overline{q}_y = \beta - \overline{u}_{yy} + L_d^{-2}(\overline{u} - \overline{u}_2), \tag{1}$$

where $f = 2\Omega \sin \lambda$ is the Coriolis parameter, $\beta = (2\Omega \cos \lambda)/R$ is the gradient of the Coriolis parameter, $\overline{u}$ and $\overline{u}_2$ are the upper-layer and lower-layer zonal-wind profiles, respectively, and $L_d = c/f$ is the radius of deformation. The radius of deformation indicates the horizontal distance to which pressure anomalies spread (deform) under the influence of gravity before they are sustained by the Coriolis force. Two vortices separated by more than a radius of deformation tend not to see each other. Further details on this important length scale may be found in Pedlosky (1987) and Gill (1982).

The basic-state assumptions for various published $1\frac{1}{2}$-layer models are listed in Table I. Choosing a basic state is akin to fixing a second parameter of the model besides L_d and needs to be justified. Here we single out the basic state introduced by Dowling (1993) in which the upper-layer zonal wind satisfies $\overline{u} = \overline{q}_y L_d^2$ and the lower-layer zonal wind satisfies $\overline{u}_2 = (\beta - \overline{u}_{yy})L_d^2$. This basic-state was first discovered empirically using the vortex-tube stretching analysis of the Voyager cloud top wind data for Jupiter's Great Red Spot and White Oval BC by Dowling and Ingersoll (1989). One feature that makes this basic state significant is that it violates all of the commonly known stability theorems, i.e., barotropic, Charney–Stern, Fjørtoft. Discussions between the author and G. Flierl led subsequently to the realization that this basic state is neutrally stable with respect to Arnol'd's second stability criterion. Benzi *et al.* (1982) applied Arnol'd's nonlinear stability analysis to the $1\frac{1}{2}$-layer quasigeostrophic model. Dowling (1995) reviews this theory in the context of jovian atmospheric dynamics, and includes the linear (small perturbation) analysis that yields the following two sufficient stability criteria:

A $1\frac{1}{2}$-layer quasigeostrophic zonal wind profile $\overline{u}(y)$ is stable if there exists a constant α such that, for all y, either

$$\text{Case 1: } \frac{\overline{u} - \alpha}{\overline{q}_y} < \min\left\{\frac{1}{k^2 + l^2 + L_d^{-2}}\right\} = 0 \tag{2}$$

or

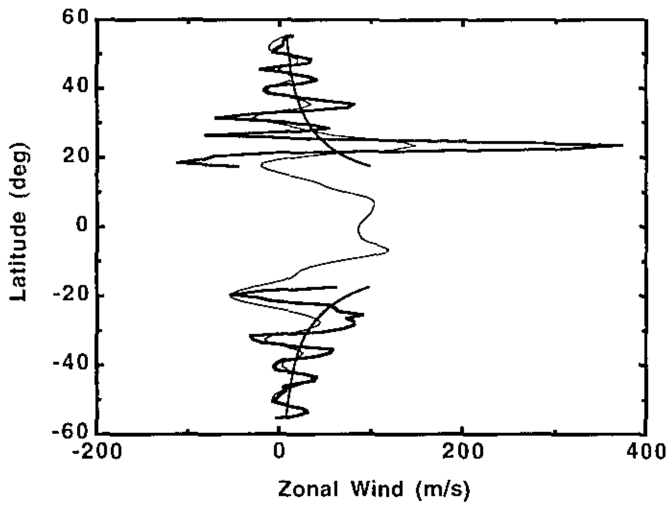

FIG. 1. Zonal wind versus planetographic latitude on Jupiter from a simple model ($1\frac{1}{2}$ layer, quasigeostrophic) with no free parameters. The thin wavy curve is the cloud-top zonal-wind profile (Limaye 1986). The smooth curve is βL_d^2. The thick curve shows the deep zonal-wind assuming neutral stability with respect to Arnol'd's second stability criterion, using the gravity wave speed $c = 454$ msec^{-1} as determined from the HST observations of Shoemaker–Levy 9 impact sites. The deep profile is not shown equatorward of 17° because quasigeostrophic theory breaks down around the equator. There is a 10% error from the formal uncertainty in c; the fine-scale structure is probably not real. Predictions are that the westward jets remain relatively unchanged underneath the cloud tops but that the eastward jets increase in strength by 50–100% over their speeds at the cloud tops.

$$\text{Case 2: } \frac{\bar{u} - \alpha}{\bar{q}_y} > \text{Max}\left\{\frac{1}{k^2 + l^2 + L_d^{-2}}\right\} = L_d^2. \quad (3)$$

Case 1 was first derived by Fjørtoft (1950) and reduces to the statement that the flow is stable if there is a reference frame shift α in which the zonal wind and the potential-vorticity gradient are always of opposite sign (note from Eq. (1) that $\bar{q}_y$ is Galilean invariant). By taking $\alpha <$ Min$\{\bar{u}\}$ and $\alpha >$ Max$\{\bar{u}\}$, the Charney–Stern stability criterion is recovered, which states that the flow is stable if $\bar{q}_y$ does not change sign. This in turn includes as a special case the barotropic stability criterion, which states that the flow is stable if $\beta - \bar{u}_{yy}$ does not change sign, applicable when $\bar{u} = \bar{u}_2$. Last, one recovers Rayleigh's classic inflection-point stability criterion, that the flow is stable if $\bar{u}_{yy}$ does not change sign, applicable for the nonrotating case ($\Omega = 0$). Each of these stability theorems deriving from Eq. (2) are discussed in detail in the standard dynamic meteorology textbooks such as Gill (1982), Holton (1992), and Pedlosky (1987). In contrast, Case 2 is not discussed in current textbooks and as a consequence is not well known.

Both of Arnol'd's two stability criteria provide only sufficient conditions for stability and no information when they are violated. Stamp and Dowling (1993) studied Case 2 and showed numerically that the condition $\bar{u} = \bar{q}_y L_d^2$ describes a marginal stability boundary for sinusoidal shear flows. Note that in problems where the horizontal domain is not large in comparison with the radius of deformation, for example narrow-channel flow, the right-hand side of Eq. (3) equals Max$\{1/(k^2 + l^2 + L_d^{-2})\}$, as discussed by McIntyre and Shepherd (1987), but for Jupiter this reduces to L_d^2 (small meridional wavenumbers l are possible because of the coherence across jets of long Rossby waves afforded by the Case 2 basic state).

Figure 1 shows the results. Latitude is planetographic and an eastward jet is one that extends to the right in the figure. Because $\bar{u}_2$ derives from the second derivative of $\bar{u}$, care must be taken to properly smooth the Limaye (1986) cloud-top profile in order to reduce observational noise and small-scale variations that are not covered by quasi-geostrophic theory. We have elected to use a Savitzky–Golay filter, as discussed by Press *et al.* (1992). This filter was originally designed for smoothing noisy spectral-line data, and has the property that it does not degrade the height and width of a line in the way that a uniform (boxcar) average does. It works particularly well when the averaging window is approximately the FWHM of the line. Observations of planetary zonal winds are an ideal candidate for this type of smoothing because the jet widths are quite regular. The thin wavy curve in Fig. 1 is $\bar{y}$ obtained by first using a cubic spline interpolation to sample Limaye's data table at 11 equally spaced points in latitude in the range ±2.5° centered around the latitude of interest and then averaging using the following weights: −0.084, 0.021, 0.103, 0.161, 0.196, 0.207, 0.196, 0.161, 0.103, 0.021, −0.084. To the eye, this procedure closely preserves the jet widths and heights in the original $\bar{u}$ profile while significantly reducing the noise when calculating $\bar{u}_{yy}$. Other Savitsky–Golay weighting schemes that preserve higher-order moments were also tried and generally gave similar but somewhat noisier results. The heavy curve in Fig. 1 is $\bar{u}_2 = (\beta - \bar{u}_{yy})L_d^2$, where L_d is given by (454 msec^{-1})/f, and $\bar{u}_{yy}$ is computed by finite differencing the smoothed $\bar{u}$ profile (thin line in Fig. 1) using $\Delta\lambda = 1°$ for each derivative and taking into account Jupiter's oblate-spherical map factors. The curve βL_d^2 is included for comparison.

No smoothing was done on the $\bar{u}_2$ profile in Fig. 1. We do not mean to suggest, however, that what looks like noise in this profile is anything other than noise. In particular, we doubt the reality of the extra westward jet at $\lambda = 27°$ planetographic latitude that has appeared just north of the very strong eastward jet at $\lambda = 24°$ or the noisy detail in the eastward jet at $\lambda = -25°$. We do believe that the very strong eastward jet at $\lambda =$ 24° does get significantly stronger below the cloud tops, but perhaps by less than is indicated by the heavy curve in Fig. 1. A 10% error in $\bar{u}_2$ comes from the 20 msec^{-1} formal error in the HST-SL9 value for c. Other errors are difficult to estimate because of the crudeness of the model, but are probably not small.

The fact that the $\bar{u}$ and $\bar{u}_2$ profiles in Fig. 1 match at high latitudes, especially over the range −40° to −58°, lends credibility to our procedure for calculating $\bar{u}_2$. Note that a quasi-sinusoidal cloud-top jet with north–south wavelength L that is neutrally stable with respect to Arnol'd's second stability criterion satisfies $\bar{u}_2 \approx \beta L_d^2 + B_u \bar{u}$, where $B_u = (2\pi L_d/L)^2$ is the Burger number. Thus, at high latitudes where βL_d^2 is small a match between $\bar{u}$ and $\bar{u}_2$ implies $B_u \approx 1$. Orsolini and Leovy (1994) found that $B_u = 1$ minimizes baroclinic instability in Jupiter-like jets, a point that should be pursued in light of these results (note that Orsolini and Leovy define the Burger number to be the inverse of the more common definition used here).

Two additional points should be made about the way in which we have fixed the parameters in our model in order to arrive at a model with no free parameters. First, our assumption that the gravity wave speed c is approximately a constant with respect to latitude may be too simplistic. There is no intrinsic reason why this parameter could not vary with latitude, since the vertical stratification is sensitive to cloud moisture and structure, and Jupiter is a heterogeneous planet. We assume a constant value here only because that is an assumption made in quasigeostrophic theory and we have data only for the SL9 impact latitude. Note that the deep-wind profile in this paper scales as c^2, so that a 20% change in c corresponds to a 40% change in $\bar{u}_2$. Second, the idea that Jupiter's winds are neutrally stable with respect to Arnol'd's second stability theorem is new and the corresponding basic-state is not well studied. For discussion of other basic states that may be relevant to Jupiter consult the special issue of the journal *Chaos* (1994, Vol. 4(2); Nezlin, Ed.); in particular the reader may wish to compare and contrast the papers by Busse, Dowling, Marcus and Lee, and Rhines.

Given the above qualifications, the conclusions from this work are that the westward jets remain relatively unchanged underneath Jupiter's cloud tops but that the eastward jets increase in strength by 50–100% over their speeds at the cloud tops. Since the lower layer in this model represents the barotropic component of Jupiter's circulations, the heavy curve in Fig. 1 presumably indicates the winds speeds in Jupiter' interior below the H_2O cloud. This is ostensibly a more appropriate zonal-wind profile for interior models to target than the NH_3 cloud-top profile. A future mission to Jupiter with multi-latitude entry probes should sample the vertical structure of both eastward and westward jets, and should take these predictions into account during the planning stage. Much future work is needed in this area. For example, global multi-layer numerical models should be used to test the validity of the $1\frac{1}{2}$-layer results, and a theory that predicts the vertical structure of Jupiter's equatorial jet is needed.

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